BASIC POLYNOMIALS

Common terms

A **monomial** is an expression that contains only **one term**, e.g. $5x,\ x^2,\ 2ab,\ 5a^2b^3$

A **binomial** is an expression that contains **two terms** added or subtracted, e.g.:

$$x + y, \qquad 3a - 2b, \qquad x^2 + 1, \qquad 3y - 4$$

A **trinomial** is an expression that contains **three terms** added or subtracted, e.g.:

$$x^2 - 5x + 6, \qquad x + y - 4, \qquad 4x^2 - 2xy + y^2, \qquad m + n - p$$

A **quadratic trinomial** is a trinomial of the form $ax^2 + bx + c$ (where $a \neq 0, b \neq 0, c \neq 0$); a is the coefficient of x^2, b is the coefficient of x, and c is the constant term.

Standard results

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

$(x + m)(x + n) = x^2 + (m + n)x + mn$

In each of these results, the expression of the left-hand side has been **expanded** to obtain the expression on the right.

If we start with the expression on the right-hand side, then we can **factorise** it to obtain the usually shorter form on the left.

Example 3

Expand and simplify each expression.

(a) $(x+2)(x+3)$ **(b)** $(3x-2)(2x+3)$ **(c)** $(2y+5)^2$

(d) $(3x-4)(3x+4)$ **(e)** $(x+2)(x^2-5x+6)$ **(f)** $(x-1)(x+2)(x+3)$

Solution

(a) $(x+2)(x+3)$
$= x(x+3) + 2(x+3)$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

(b) $(3x-2)(2x+3)$
$= 6x^2 + 9x - 4x - 6$
$= 6x^2 + 5x - 6$

(c) $(2y+5)^2$
$= 4y^2 + 20y + 25$

(d) $(3x-4)(3x+4)$
$= 9x^2 - 16$

(e) $(x+2)(x^2-5x+6)$
$= x(x^2-5x+6) + 2(x^2-5x+6)$
$= x^3 - 5x^2 + 6x + 2x^2 - 10x + 12$
$= x^3 - 3x^2 - 4x + 12$

(f) $(x-1)(x+2)(x+3)$
$= (x-1)(x^2+5x+6)$
$= x^3 + 5x^2 + 6x - x^2 - 5x - 6$
$= x^3 + 4x^2 + x - 6$