INVESTMENTS AND LOANS

1 (a) Julian deposits $1000 in an investment account that is paying a monthly interest rate of 0.3%, with the interest compounded monthly. Calculate the value of the investment after 12 months.

(b) Minh deposits $2000 in an investment account that is paying a monthly interest rate of 0.5%, with the interest compounded monthly. Calculate the value of the investment after 10 months.

(c) Ben deposits $3400 in an investment account that is paying a monthly interest rate of 0.35%, with the interest compounded monthly. Calculate the value of the investment after 20 months.

a) $A = 1{,}000\,(1+0.003)^{12} = 1{,}036.60$

b) $A = 2{,}000\,(1+0.005)^{10} = 2{,}102.28$

c) $A = 3{,}400\,(1+0.0035)^{20} = 3{,}646.08$

2 The table shows the future value of an investment of $1000, compounding yearly, at varying interest rates for different periods of time.

Future values of an investment of $1000					
Number of years	Interest rate per annum				
	1%	2%	3%	4%	5%
1	1010.00	1020.00	1030.00	1040.00	1050.00
2	1020.10	1040.40	1060.90	1081.60	1102.50
3	1030.30	1061.21	1092.73	1124.86	1157.63
4	1040.60	1082.43	1125.51	1169.86	1215.51
5	1051.01	1104.08	1159.27	1216.65	1276.28
6	1061.52	1126.16	1194.05	1265.32	1340.09

(a) Based on the information provided, what is the future value of an investment of $3000 over 4 years at 3% p.a.?
A $3374.58 **B** $3376.53 C $3278.19 D $4502.04

(b) Based on the information provided, what is the future value of an investment of $4500 over 6 years at 5% p.a.?
A $5053.28 B $5684.94 C $5360.36 **D** $6030.41

(c) Based on the information provided, what is the future value of an investment of $500 over 5 years at 4% p.a.?
A $608.33 B $607.76 C $2431.02 D $2433.30

(d) Based on the information provided, what is the future value of an investment of $700 over 3 years at 2% p.a.?
A $7426.30 B $7428.47 **C** $742.85 D $742.63

a) $A = 1{,}125.51 \times \frac{3{,}000}{1{,}000} = 3{,}376.53$

b) $A = 1{,}340.09 \times \frac{4{,}500}{1{,}000} = 6{,}030.41$

c) $A = 1{,}216.65 \times \frac{500}{1{,}000} = 608.33$

d) $A = 1{,}061.21 \times \frac{700}{1000} = 742.85$

INVESTMENTS AND LOANS

some important item ☺

3 Brian and Faye plan to have $25 000 in an investment account in 10 years time to pay for ~~a cruise.~~ The interest rate for the account will be fixed at 4.2% per annum, compounded monthly. How much do they need to deposit into the account to achieve this goal? Round your answer to the next dollar.

$A = P(1+r)^n$ so $P = \frac{A}{(1+r)^n} = \frac{25,000}{(1+\frac{0.042}{12})^{10\times12}} = 16,438$

4 What amount must be invested now at 6% per annum, compounded monthly, so that in 4 years it will have grown to $50 000?

$A = P(1+r)^n$ so $P = \frac{A}{(1+r)^n} = \frac{50,000}{(1+\frac{0.06}{12})^{4\times12}} = 39,354.92$

5 The table shows the compounded values of $1.

Future values of the compounded values of $1						
Period	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	1.010	1.020	1.030	1.040	1.050	1.060
2	1.020	1.040	1.061	1.082	1.103	1.124
3	1.030	1.061	1.093	1.125	1.158	1.191
4	1.041	1.082	1.126	1.170	1.216	1.262
5	1.051	1.104	1.159	1.217	1.276	1.338
6	1.062	1.126	1.194	1.265	1.340	1.419
7	1.072	1.149	1.230	1.316	1.407	1.504
8	1.083	1.172	1.267	1.269	1.477	1.594

Use this table to calculate the value of the following investments.

(a) Stuart invests $2500 for 4 years at an interest rate of 3% per half year compounded half yearly.
(b) Karina invests $5600 for 2 years at an interest rate of 1% per quarter, compounded quarterly.
(c) Aaliyah invests $3000 for 5 years at an interest rate of 5% per annum compounded yearly.

a) $4 \times 2 = 8$ periods. so $A = 1.267 \times 2,500 = 3,167.50$

b) $2 \times 4 = 8$ periods so $A = 1.083 \times 5,600 = 6,064.80$

c) 5 periods $A = 3000 \times 1.276 = 3,828$

6 Following is a table of future value interest factors for a $1 contribution to an annuity.

Future value interest factors					
Period	Interest rate per period				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.0101	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019

Answer the following questions using this table.

(a) A certain annuity involves making equal contributions of $1000 into an account every 4 months for 2 years at an interest rate of 6% per annum. The future value of this annuity is:
A $6308.10 B $6468.40 C $6633.00 D $4121.60

(b) A certain annuity involves making equal contributions of $5000 into an account every 6 months for 2 years at an interest rate of 4% per annum. The future value of this annuity is:
A $10 100.00 B $10 200.00 C $21 232.50 D $20 608.00

(c) A certain annuity involves making equal contributions of $500 into an account every year for 6 years at an interest rate of 5% per annum. The future value of this annuity is:
A $3316.50 B $6633.00 C $3400.95 D $6801.90

a) $\frac{12}{4} \times 2 = 6$ periods. interest per period is $\frac{0.06}{3} = 0.02$

So FV $= 6.3081 \times 1{,}000 = 6{,}308.10$ [A]

b) $\frac{12}{6} \times 2 = 4$ periods interest per period $= \frac{4\%}{2} = 2\%$

So FV $= 4.1216 \times 5{,}000 = 20{,}608$ [D]

c) 6 periods interest rate is 5%

So FV $= 6.8019 \times 500 = 3{,}400.95$ [C]

8 The table gives the present value interest factors for an annuity of \$1 per period, for various interest rates r and number of periods N.

Present value interest factors					
N	Interest rate per period (as a decimal) (r)				
	0.0025	0.005	0.0075	0.008	0.009
71	64.9814	59.6412	54.8929	54.0075	52.2966
72	65.8169	60.3395	55.4769	54.5710	52.8212
73	66.6502	61.0343	56.0564	55.1299	53.3411
74	67.4815	61.7257	56.6317	55.6845	53.8564
75	68.3108	62.4137	57.2027	56.2346	54.3671
76	69.1379	63.0982	57.7694	56.7803	54.8732

Use the table to answer the following questions.

(a) What is the present value of an annuity of \$200 per month for 71 months if the interest rate is 0.75% per month?
A \$11095.38 B \$10978.58 C \$11928.24 D \$10801.50

(b) What is the present value of an annuity of \$150 per month for 74 months if the interest rate is 0.8% per month?
A \$8078.46 B \$8268.49 C \$8352.68 D \$8435.19

(c) A loan of \$20000 is to be repaid in equal monthly instalments over 6 years. The interest rate is 10.8% per annum. What are the monthly repayments, rounded to the next dollar?
A \$379 B \$367 C \$361 D \$375

(d) A loan of \$8000 is to be repaid in equal monthly instalments over 6 years. The interest rate is 9% per annum. What are the monthly repayments, rounded to the next dollar?
A \$146 B \$143 C \$140 D \$145

a) 71 periods interest rate 0.75%/month

$PV = 54.8929 \times 200 = 10,978.58$ B

b) 74 periods interest rate 0.8%/month

$PV = 55.6845 \times 150 = 8,352.68$

c) $6 \times 12 = 72$ periods interest rate is $\frac{10.8\%}{12} = 0.9\%$/month

$20,000 = 52.8212 \times \text{payment}$ so $\text{payment} = \frac{20,000}{52.8212} = 378.64$ A

d) $6 \times 12 = 72$ periods interest rate is $\frac{9\%}{12} = 0.75\%$/month

$8,000 = 55.4769 \times \text{payment}$ so $\text{payment} = \frac{8,000}{55.4769} = 144.20$

closest is D

9 The table gives the contribution per period for an annuity with a future value of $1 at different interest rates and different periods of time.

Contributions per period for an annuity with a future value of $1						
Number of periods	Interest rate (% per period)					
	0.25%	0.5%	0.75%	1%	1.25%	1.5%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0.3325	0.3317	0.3308	0.3300	0.3292	0.3284
6	0.1656	0.1646	0.1636	0.1625	0.1615	0.1605
9	0.1100	0.1089	0.1078	0.1067	0.1057	0.1046
12	0.0822	0.0811	0.0800	0.0788	0.0778	0.0767
15	0.0655	0.0644	0.0632	0.0621	0.0610	0.0599
18	0.0544	0.0532	0.0521	0.0510	0.0499	0.0488
21	0.0464	0.0453	0.0441	0.0430	0.0419	0.0409
24	0.0405	0.0393	0.0382	0.0371	0.0360	0.0349

Use the table to answer the following questions.

(a) Barbie and Ken need to save $100 000 over 3 years for a deposit on a new house. They make regular quarterly contributions into an investment account which pays interest at 4% p.a. How much do they need to contribute each quarter to reach this savings goal?

(b) Beatrice is saving for a deposit to buy a car. She needs to save $3000 in a year. How much must she pay into an investment account each month, if the interest is 3% p.a., calculated monthly, to reach this goal?

(c) Danh is starting a superannuation fund. He wishes to have $1 000 000 in the account after 12 years. Interest is 3% p.a. If he makes a deposit to this account every six months, how much should he deposit, rounded to the next dollar, to be sure that he reaches his goal?

a) $3 \times 4 = 12$ periods. interest per period $= \frac{4\%}{4} = 1\%$ / quarter.

So Contribution $= 100{,}000 \times 0.0788 = 7{,}880$

b) 12 periods interest per period $= \frac{3\%}{12} = 0.25\%$ / month

So Contribution $= 3{,}000 \times 0.0822 = 246.60$

c) $12 \times 2 = 24$ periods interest per period $= \frac{3\%}{2} = 1.5\%$ / half year

So Contribution $= 1{,}000{,}000 \times 0.0349 = 34{,}900$

11 What is the effective annual interest rate for a loan advertised as:

(a) 4% p.a. compounded quarterly (b) 4% p.a. compounded monthly
(c) 4% p.a. compounded daily (use 365 days in a year)?

Effective annual interest rate $= \left(1+\frac{r}{n}\right)^n - 1$

a) $E_{air} = \left(1+\frac{0.04}{4}\right)^4 - 1 = 4.06\%$

b) $E_{air} = \left(1+\frac{0.04}{12}\right)^{12} - 1 = 4.074\%$

c) $E_{air} = \left(1+\frac{0.04}{365}\right)^{365} - 1 = 4.081\%$

14 The home loan from Building Society N is advertised with an interest rate of 5.4% compounded monthly. Credit Union T advertises their home loan with an interest rate of 5.35% compounded daily. If there are no other fees involved, which financial institution offers the better deal?

Building Society N

$E_{air} = \left(1+\frac{0.054}{12}\right)^{12} - 1 = 5.5356\%$

Credit Union T

$E_{air} = \left(1+\frac{0.0535}{365}\right)^{365} - 1 = 5.4953\%$

So the Credit Union T has the lowest rate, it's the best deal.