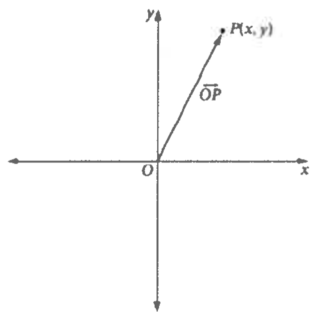
VECTORS IN TWO DIMENSIONS

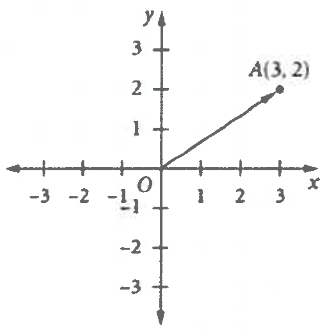
Position vectors on the Cartesian plane

A position vector is a vector drawn with its tail at O, the origin. Position vectors are use to represent points by a distance and a direction rather than two numbers (the coordinates).

The position vector of any point $P(x, y)$ relative to a fixed origin O on the Cartesian plane is uniquely specified by the vector $\overrightarrow{OP}$, as shown in the diagram. That is, the vector $\overrightarrow{OP}$ represents the position vector of point $P(x, y)$ relative to O.



For example, the diagram at right shows the position vector of point $A(3, 2)$ relative to O. The position vector is $\overrightarrow{OA}$, where A has coordinates (3, 2).



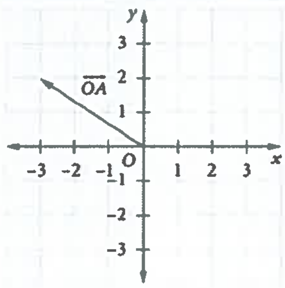
Example 6

Draw the following position vectors on the Cartesian plane.

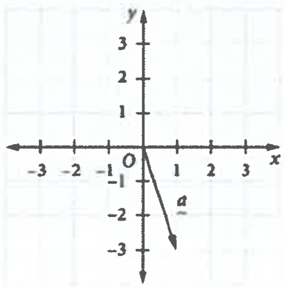
(a) $\overrightarrow{OA}$, where A is (−3, 2) **(b)** $\underset{\sim}{a}$, the position vector of (1, −3)

Solution

(a) Vector is drawn from the initial point at the origin O to the given coordinates.



(b) Vector is drawn from the initial point at the origin O to the given coordinates.



A position vector can be represented by a coordinate pair (a, b). This represents the vector that is a units from O in the positive x direction, and b units from O in the positive y direction. This can be represented as a column vector $\begin{pmatrix} a \\ b \end{pmatrix}$, $\begin{bmatrix} a \\ b \end{bmatrix}$ or by the coordinates (a, b). Any vector that is equivalent to a translation of a units in the positive x-direction and b units in the positive y-direction can be represented in this way.

If A has coordinates (3,2), then the position vector $\overrightarrow{OA}$ can be represented by the column vector $\binom{3}{2}$, i.e. $\overrightarrow{OA} = \binom{3}{2}$.

VECTORS IN TWO DIMENSIONS

The magnitude of a position vector

The magnitude of a position vector $\overrightarrow{OA} = \binom{a}{b}$ is noted $|\overrightarrow{OA}|$

It is a scalar that can be calculated using Pythagoras's theorem: $|\overrightarrow{OA}| = \sqrt{a^2 + b^2}$

For example, the magnitude of the position vector $\overrightarrow{OA} = \binom{3}{2}$ is $|\overrightarrow{OA}| = \sqrt{3^2 + 2^2} = \sqrt{13}$ units.

Example 7

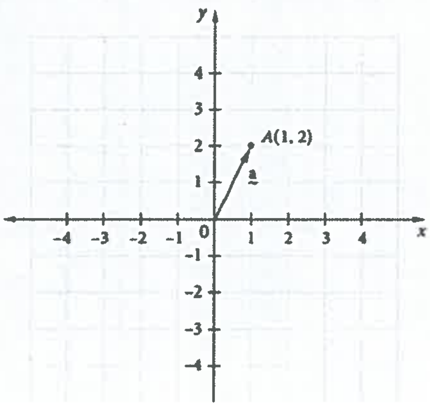
Draw the following position vectors on the Cartesian plane.

(a) $\underset{\sim}{a}$, the position vector of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. (b) $\overrightarrow{OB} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Solution

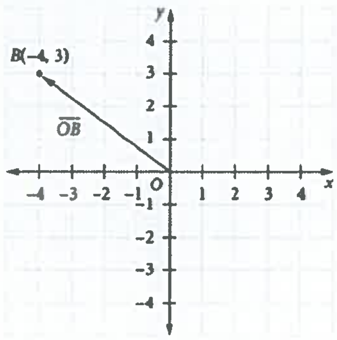
(a) $\underset{\sim}{a}$ represents a vector which is 1 unit from O in the positive x direction and 2 units from O in the positive y direction.

Vector is drawn from the origin to the point (1, 2).



(b) $\overrightarrow{OB}$ represents a vector which is −4 units from O in the positive x direction (4 units in the negative x direction) and 3 units from O in the positive y direction.

Vector is drawn from the origin to the point (−4, 3).

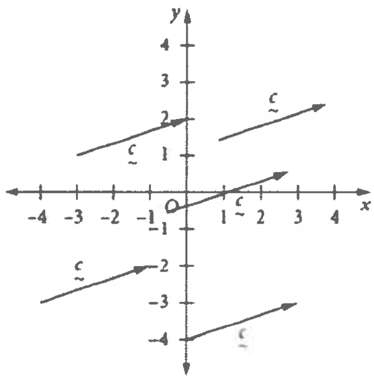


Equal vectors

For two vectors to be equal, they need to have the same magnitude and the same direction.

Although the position vector of the point $A(3,1)$ starts at the origin, the coordinate pair (3,1) can be used to represent any vector whose head is three units across and one unit up from its tail. However, none of the other vectors can be called position vectors .

Vectors will also be equal if and only if they are expressed using the same coordinates. For example, the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ will only be equal to the vector $\begin{bmatrix} c \\ d \end{bmatrix}$ if $a = b$ and $c = d$



VECTORS IN TWO DIMENSIONS

Example 8

Given $\underset{\sim}{d} = (2, 5)$, specify an ordered pair for each of the following. **(a)** $2\underset{\sim}{d}$ **(b)** $-\underset{\sim}{d}$

Solution

(a) As this vector has twice the magnitude, each coordinate will be multiplied by 2.
$2\underset{\sim}{d} = (4, 10)$.

(b) As this vector has the same magnitude but in the opposite direction to $\underset{\sim}{d}$, each coordinate must also be negated (multiplied by -1).
The vector $-\underset{\sim}{d}$ is represented by the ordered pair $(-2, -5)$.

Example 9

Given $\underset{\sim}{e} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$, specify a column vector for each of the following. **(a)** $\frac{1}{2}\underset{\sim}{e}$ **(b)** $-3\underset{\sim}{e}$

Solution

(a) As this vector has half the magnitude, each coordinate is multiplied by $\frac{1}{2}$.
The vector $\frac{1}{2}\underset{\sim}{e}$ is represented by the column vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

(b) As this vector has three times the magnitude but in the opposite direction to $\underset{\sim}{e}$, each coordinate must also be negated (multiplied by -3).
The vector $-3\underset{\sim}{e}$ is represented by the column vector $\begin{pmatrix} -12 \\ 18 \end{pmatrix}$.