Express each of the following as sums or differences:

 $2 \sin 4\theta \cos 2\theta$ 

 $3 2\cos 3A\cos 5A$ 

 $4 \cos 4A \sin 2A$ 

5  $\sin(\theta + \alpha)\cos(\theta - \alpha)$  6  $2\cos(45^{\circ} + A)\sin(45^{\circ} - A)$  7  $\cos(2\theta + \alpha)\cos(2\theta - \alpha)$ 

15  $2\sin(\theta+\phi)\cos(\theta-\phi)$ 14 cos 75° cos 45°

16  $\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$ 

**17**  $\sin 100^{\circ} \sin 130^{\circ}$  **18**  $2 \sin 3\theta \cos \theta$ 

19  $\cos(\theta + 2\phi) \sin(2\theta + \phi)$ 

Express the following as products:

23  $\sin 3x - \sin x$  24  $\sin (x + \alpha) - \sin x$  25  $\cos (x + h) - \cos x$  26  $\sin (\theta + \alpha) + \sin (\theta - \alpha)$  27  $\cos \left(\frac{\theta + \alpha}{2}\right) + \cos \left(\frac{\theta - \alpha}{2}\right)$  28  $\cos (A + B + C) - \cos (A - B + C)$ 

**36**  $\sin(2A+2B) - \sin(2A-2B)$  **37**  $\sin 165^{\circ} - \sin 105^{\circ}$  **38**  $\sin \theta - \cos \phi$ 

**39**  $\cos 75^{\circ} - \cos 45^{\circ}$  **40**  $\sin 50^{\circ} + \cos 20^{\circ}$  **41**  $\sin (A - B) - \sin A$ 

Prove the following results.

43 
$$\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \tan \theta$$

$$45 \frac{\cos x - \cos 3x}{\sin x - \sin 3x} = -\tan 2x$$

44 
$$\frac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\frac{x+y}{2}\right)$$

46 
$$\frac{\sin 2A - \sin 2B}{\sin 2A + \sin 2B} = \frac{\tan(A-B)}{\tan(A+B)}$$

Prove the following results.

**54** 
$$2\cos 37.5^{\circ} \sin 7.5^{\circ} = \frac{\sqrt{2}-1}{2}$$

56 
$$\sin 10^{\circ} + \cos 40^{\circ} = \sin 70^{\circ}$$

55 
$$\sin 25^{\circ} \sin 35^{\circ} - \sin 20^{\circ} \sin 10^{\circ} = \frac{\sqrt{3} - 1}{4}$$

$$57 \frac{\sin 48^{\circ} + \sin 12^{\circ}}{\cos 48^{\circ} + \cos 12^{\circ}} = \frac{\sqrt{3}}{3}$$

Prove the following results.

$$\frac{\sin\theta + \sin 7\theta}{\sin 3\theta + \sin 5\theta} = 2\cos 2\theta - 1$$

**61** 
$$\frac{\sin(\theta + \phi) - \sin(\theta - \phi)}{\cos(\theta + \phi) - \cos(\theta - \phi)} = -\cot\theta$$

Prove the following results.

**65** If  $\alpha + \beta + \gamma = \pi$ , show that  $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4 \sin \alpha \sin \beta \sin \gamma$ .