# FUNCTIONS - CHAPTER REVIEW

**1** State the largest possible domain for the following functions:

**(a)** $f(x)=\sqrt{x-1}$ **(b)** $f(x)=\dfrac{1}{x^2-4}$ **(c)** $f(x)=\sqrt{25-x^2}$ **(d)** $f(x)=|x|$

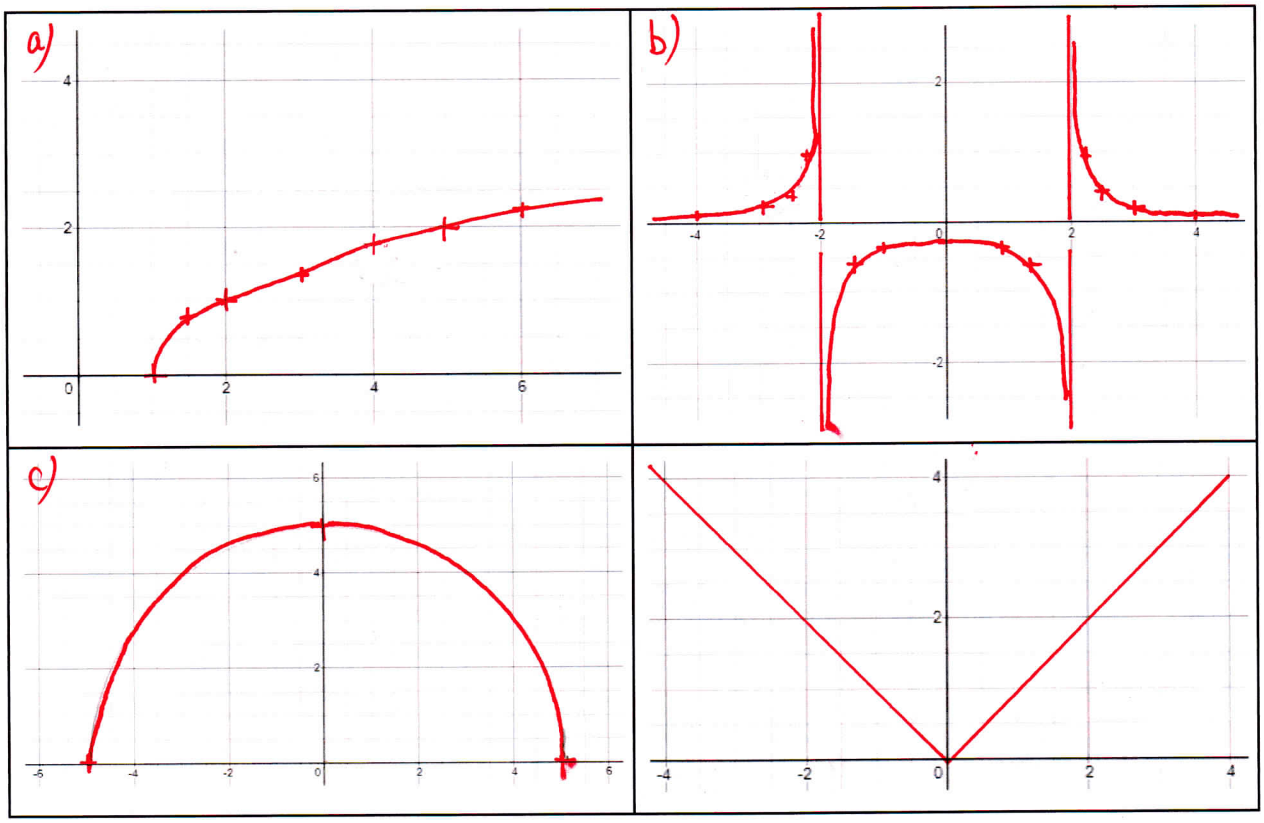
a) $(x-1)\geqslant 0$ , i.e $x\geqslant 1$ so the natural domain is $[1,+\infty)$

b) $x^2-4\neq 0$ , or $x\neq \pm 2$ so the natural domain is $\mathbb{R}-\{-2,2\}$

c) $25-x^2\geqslant 0 \iff x^2\leqslant 25$ so $-5\leqslant x\leqslant 5$ , or $[-5,5]$

d) $\mathbb{R}$

**2** Sketch the graph of each function given in question **1**.




**3** If $g(x)=x^4-x^2+1$, show that $g(x)$ is an even function.

$g$ is an even function if for all $x$, $g(-x)=g(x)$.

$g(-x)=(-x)^4-(-x)^2+1 = x^4-x^2+1 = g(x)$

So it's true that for all $x$, $g(-x)=g(x)$.

$\therefore$ $g$ is an even function.

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**7** Is the function $y = x^3 - 1$ even, odd or neither?

The function would be even if $f(-x) = f(x)$ for all $x$, and odd if $f(-x) = -f(x)$ for all $x$.

$f(-x) = (-x)^3 - 1 = -x^3 - 1$ which is different of $f(x)$ and of $-f(x)$.

so it's neither.

**9** The equation of a circle is $x^2 + y^2 - 2x - 2y - 23 = 0$.

**(a)** Find the circle's centre and radius.
**(b)** Calculate the distance from the point $(7, -2)$ to the centre of the circle.
**(c)** Explain why the point $(7, -2)$ is outside the circle.
**(d)** Use Pythagoras' theorem to find the length of the tangent to the circle from the point $(7, -2)$. (Note that tangent $\perp$ radius drawn to point of contact.)

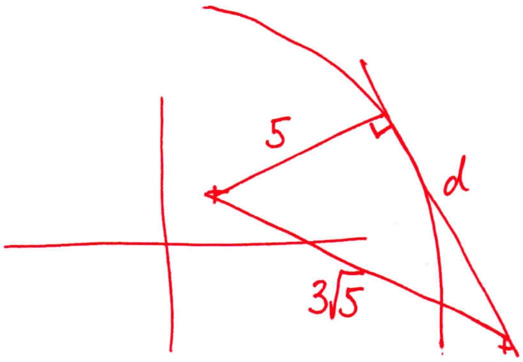
a) $x^2 + y^2 - 2x - 2y - 23 = 0 \Leftrightarrow x^2 - 2x + y^2 - 2y = 23$

$\Leftrightarrow (x-1)^2 - 1 + (y-1)^2 - 1 = 23 \Leftrightarrow (x-1)^2 + (y-1)^2 = 25 = 5^2$

So centre $(1,1)$, radius $5$

b) distance $= \sqrt{(1-7)^2 + (1-(-2))^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5} \simeq 6.7$

c) $3\sqrt{5} > 5$, so the distance from the centre to the point $(7,-2)$ is greater than the radius $(5)$, so the point is outside the circle.




d) So $(3\sqrt{5})^2 = d^2 + 5^2$

so $d^2 = 45 - 25 = 20$

so $d = \sqrt{20} = 2\sqrt{5}$

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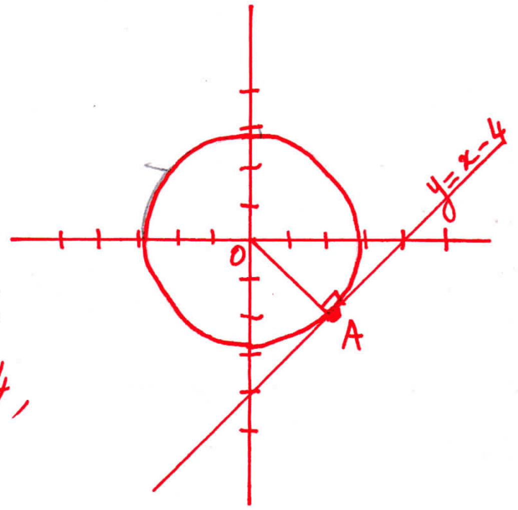
**10** Show algebraically that the line $y = x - 4$ is a tangent to the circle $x^2 + y^2 = 8$ and find the coordinates of the point of contact.

We'll try to show that the slope of the line passing through O and A is −1

First, let's find the coordinates of A.

The point $(2,-2)$ belongs to both the circle, as $2^2 + (-2)^2 = 8$, and to the line $y = x - 4$, as $(-2) = 2 - 4$.




Furthermore, the slope of the line OA is −1

∴ A belongs to both the circle and the line.

and the line OA and $y = x - 4$ are perpendicular (as the product of their gradient is −1) . ∴ the line $y = x - 4$ is tangent to the circle.

**11** Solve: **(a)** $|x + 7| = 11$ **(b)** $|3x - 4| \geq 5$

a) $|x+7| = 11 \iff x+7 = \pm 11 \iff x = \pm 11 - 7$

so $x = 4$ or $x = -18$

b) $|3x-4| \geqslant 5 \iff \begin{cases} 3x - 4 \geqslant 5 \iff x \geqslant 3 \\ \text{or } 3x - 4 \leqslant -5 \iff x \leqslant -1/3 \end{cases}$

So $x \leqslant -\frac{1}{3}$ or $x \geqslant 3$

**12** On the graph of $y = (x - 2)(x - 1)(x + 1)$, which of the following lines would you need to draw on this graph in order to solve $(x - 2)(x - 1)(x + 1) + 3 = 0$?

**A** $y = -1$ $\boxed{\textbf{B}}$ $y = -3$ **C** $y = 1$ **D** $y = 3$

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13 Solve algebraically: **(a)** $(x-3)^3 = -8$ **(b)** $(x+5)^3 = 4$ **(c)** $(x-2)^3 = 81$

a) $(x-3)^3 = -8 \iff (x-3)^3 = (-2)^3$ so $x-3=-2$ $x=1$

b) $(x+5)^3 = 4 = (4^{1/3})^3$ so $x+5 = 4^{1/3} \iff x = 4^{1/3} - 5$

c) $(x-2)^3 = 81 = 3^4 = (3^{4/3})^3$ so $x-2 = 3^{4/3}$

$x = 3^{4/3} + 2$

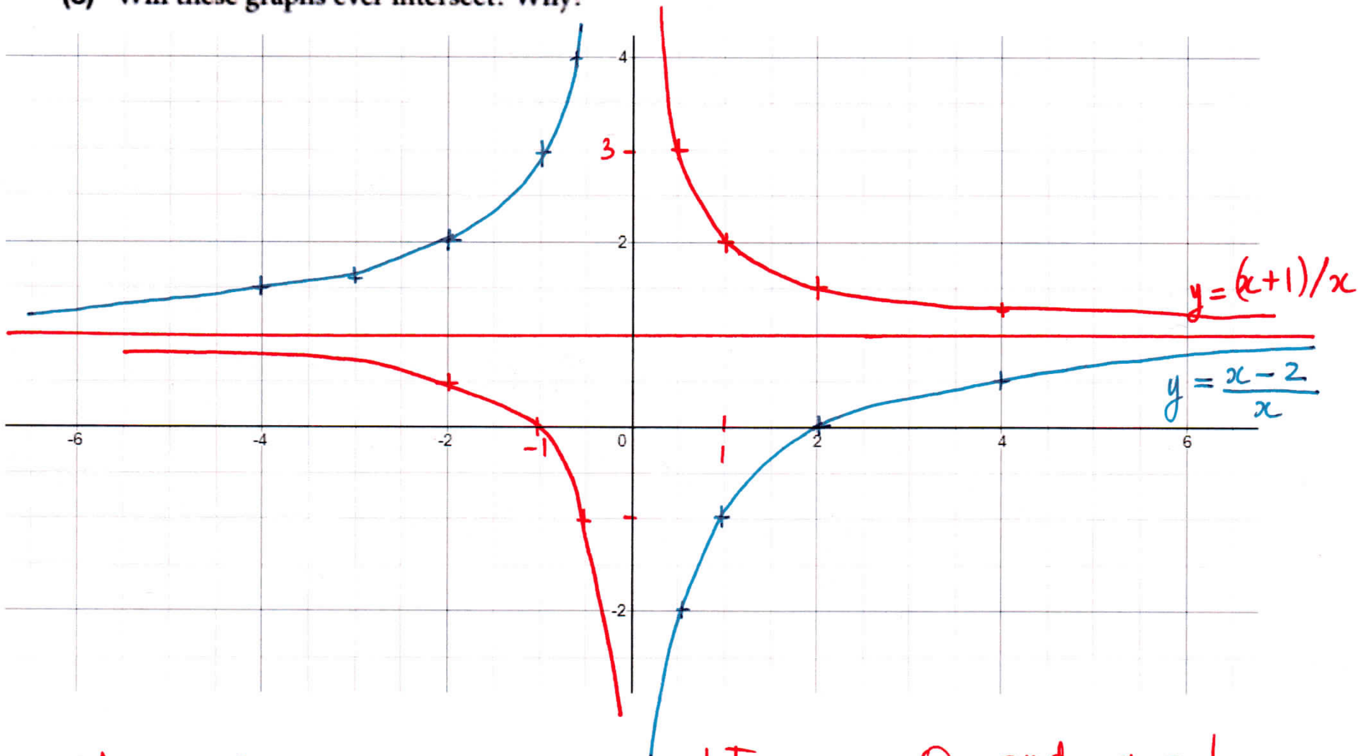
15 What are the equations of the asymptotes of the graph of $y = \frac{x}{x+3}$?

**A** $x=-3, y=-1$ **B** $x=3, y=-1$ $\boxed{\textbf{C}}$ $x=-3, y=1$ **D** $x=3, y=1$

16 **(a)** On the same diagram, draw the graphs of $y = \frac{x+1}{x}$ and $y = \frac{x-2}{x}$.

**(b)** Do these graphs have the same asymptotes?

**(c)** Will these graphs ever intersect? Why?




b) They have the same asymptotes $x=0$ and $y=1$

c) No, the graphs never intersect. [Note this would happen if for some x, $\frac{x+1}{x} = \frac{x-2}{x} \iff x+1 = x-2 \iff 1 = -2$ which is impossible $\therefore$ they don't intersect]