

FURTHER APPLICATIONS OF SERIES

- 1 Noor invests \$10 000 on the condition that the money is repaid in 12 equal quarterly instalments. If the investment earns interest at the rate of 1% per quarter, what is the amount of each instalment?

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- 2 Ava invests \$20 000 on the condition that she is repaid the money in 16 equal quarterly instalments. If the investment earns interest at the rate of 0.75% per quarter, what is the amount of each instalment?

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- 3 (a) When Nazeera started a new job, \$400 was deposited into her superannuation fund at the beginning of each month. The money was invested at 0.4% per month, compounded monthly. Let Y be the value of the investment after 360 months, when Nazeera retires. Show that $Y = 322\,142.43$.
- (b) After retirement, Nazeera withdraws \$3000 from her superannuation fund at the end of each month without making any further deposits. The account continues to earn interest at 0.4% per month. Let A_n be the amount of money left in the account n months after Nazeera's retirement.
- Show that $A_n = (Y - 750\,000) \times 1.004^n + 750\,000$.
 - For how many months after retirement will there be money left in the account?

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- 4 One year ago William and Kate borrowed \$400 000 to buy an apartment. The interest rate was 6% p.a., compounded monthly. They agreed to repay the loan over 25 years with equal monthly repayments of \$2578.
- (a) Calculate how much money William and Kate owed after their first monthly repayment.
 - (b) After making their twelfth monthly repayment, William and Kate owe \$392 870. The interest rate now increases to 9% p.a., compounded monthly. The amount A_n owing on the loan after the n th monthly repayment is now calculated using the formula $A_n = 392\,870 \times 1.0075^n - 1.0075^{n-1}M - \dots - 1.005M - M$, where M is the monthly repayment and $n = 1, 2, \dots, 288$. (Do **not** prove this formula.) Calculate the monthly repayment if the loan is to be repaid over the remaining 24 years (288 months).
 - (c) William and Kate now decide to increase their monthly repayments to \$3500. How long will it take them to repay \$392 870?
 - (d) How much money will William and Kate save over the term of the loan by making these higher monthly repayments, compared to keeping the repayments at the amount calculated in (b)?

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- 5 Which will give the better financial result after 20 years: a lump sum of \$100 000 invested at 5% p.a. compounded annually, or a monthly payment of \$600 with interest at 5% p.a. compounded monthly?

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- 6 A lottery offers a prize of \$100 000 immediately, or \$10 000 now plus \$10 000 per year for the next 11 years. You take the prize of \$100 000, keep \$10 000 to spend over the next year and invest the remaining \$90 000 as an annuity at 5% p.a. You plan to withdraw \$10 000 at the end of each year for the next 11 years.
- (a) Is this a realistic plan?
 - (b) How much money is left in your annuity after the eleventh payment?
 - (c) How much money is left in your annuity after the twelfth payment?

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- 7 Prior to retirement, Cassie deposits \$100 000 in an account which will earn interest at a rate of 2.7% p.a., paid monthly. Cassie wishes to withdraw \$300 per month.
- (a) Set up a recurrence relation that models this account and gives the balance after n payments.
 - (b) What is the balance after two withdrawals?
 - (c) Explain what will happen to the balance if the interest was 5% per annum from the start.
 - (d) What monthly payment would keep the balance at \$100 000? Calculate the answer for rates of 5% and 2.7%.

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