INEQUALITIES INVOLVING ABSOLUTE VALUE AND SQUARE ROOTS

1 $\dfrac{1}{|x|} < 3$ (Hint: The denominator is known to be non-negative.)

$\dfrac{1}{|x|} < 3 \iff 1 < 3|x| \iff |x| > \dfrac{1}{3}$

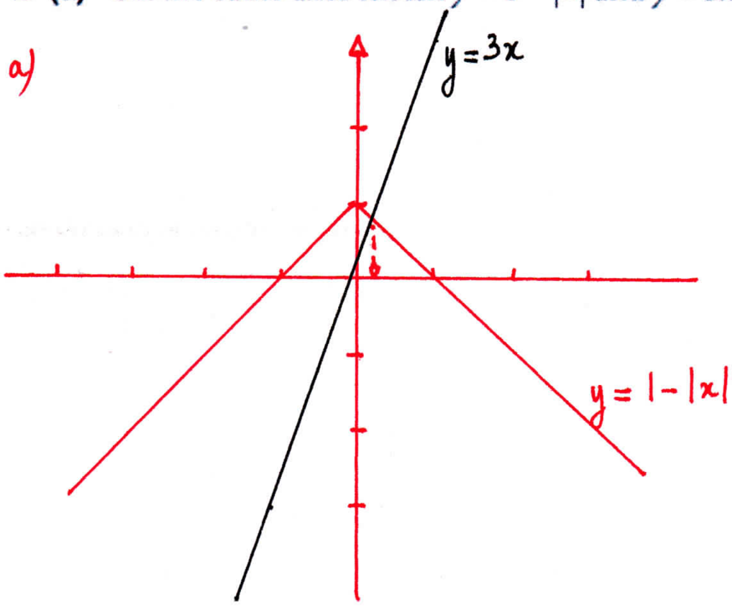
So either $x > \dfrac{1}{3}$ if $x > 0$

OR if $x < 0$ $x < -\dfrac{1}{3}$

So two interval solutions $x < -\dfrac{1}{3}$ and $x > \dfrac{1}{3}$

2 **(a)** On the same axes sketch $y = 1 - |x|$ and $y = 3x$. **(b)** Hence solve $|x| + 3x > 1$.

a)



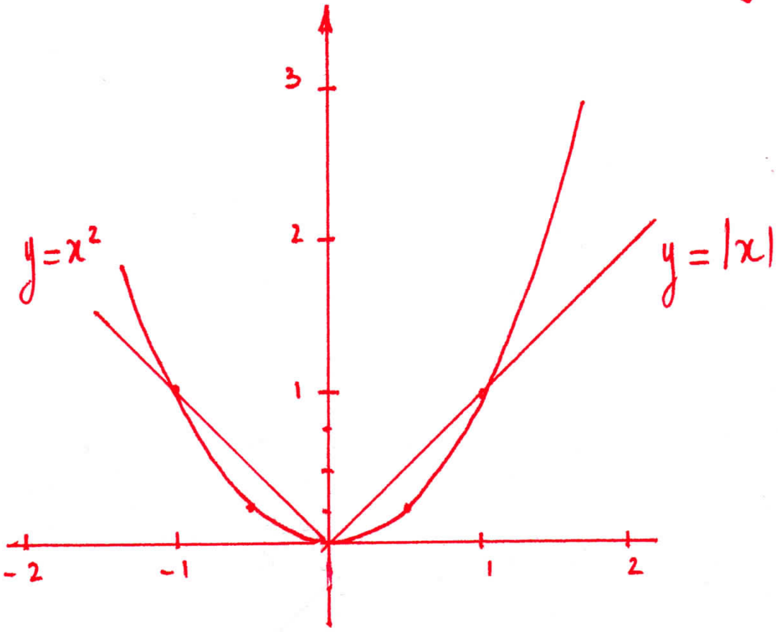
b) $|x| + 3x > 1 \iff 3x > 1 - |x|$

This situation happens when the curve $y = 3x$ is above the curve $y = 1 - |x|$.

The two curves intersect when $3x = 1 - x$ $(x>0)$ or $x = 1/4$

So when $x > \dfrac{1}{4}$, we have $|x| + 3x > 1$

4 Solve $x^2 - |x| > 0$. $\iff x^2 > |x|$



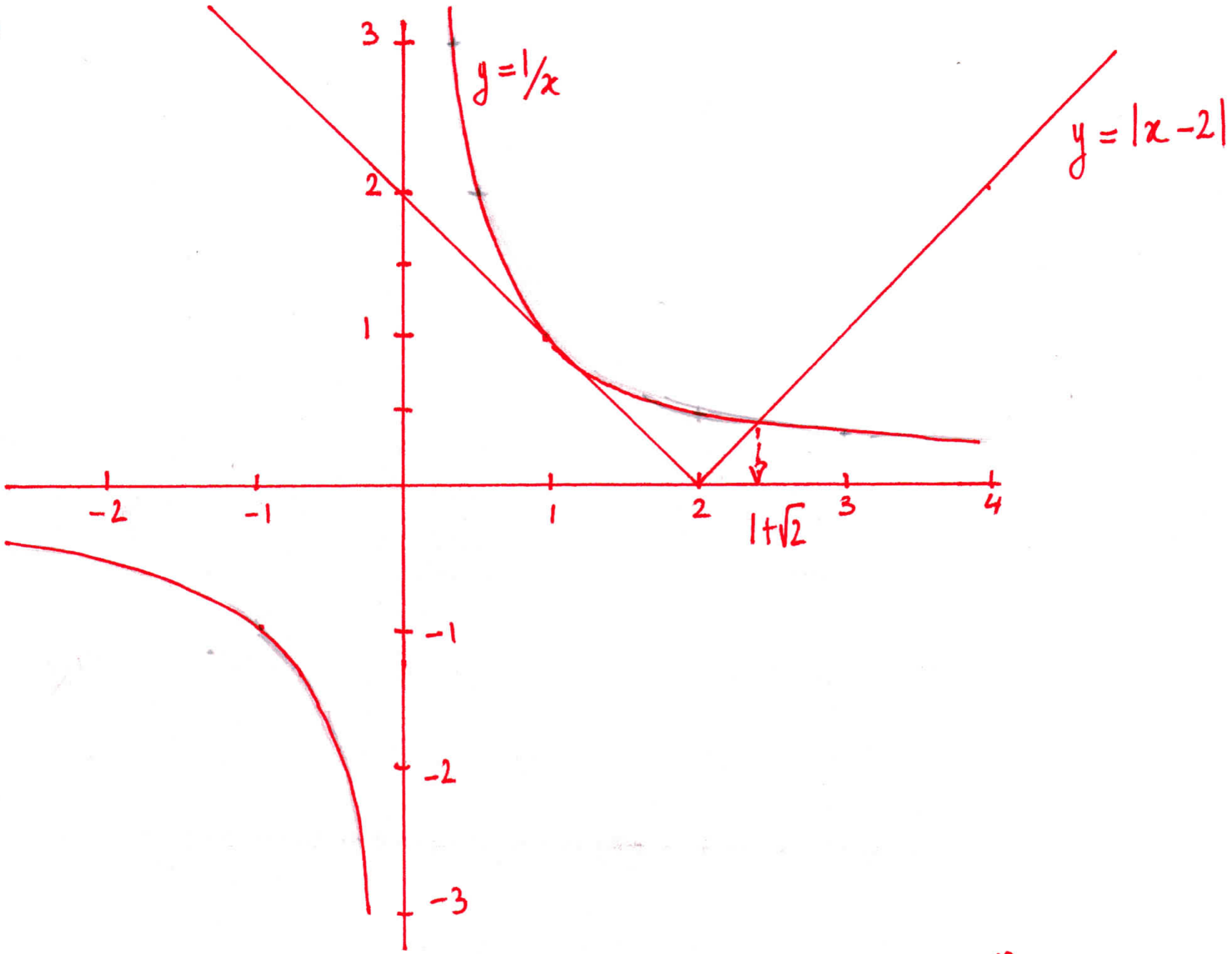
We can see that the curve $y = x^2$ is above the curve $y = |x|$ when either $x > 1$ or when $x < -1$

So the interval solutions are $x > 1$ and $x < -1$

INEQUALITIES INVOLVING ABSOLUTE VALUE AND SQUARE ROOTS

5 **(a)** On the same axes sketch $y = |x-2|$ and $y = \frac{1}{x}$. **(b)** Hence solve $|x-2| > \frac{1}{x}$.

a)



b) $|x-2| > \frac{1}{x}$ this situation occurs when the curve $y = |x-2|$ is above the curve $y = \frac{1}{x}$

This is such the case when $x < 0$ and when the two curves intercept just after $x = 2$, exactly at $x - 2 = \frac{1}{x}$

$x^2 - 2x = 1 \iff x^2 - 2x - 1 = 0$

$\Delta = 4 - 4 \times (-1) = 8$

$x_1 = \frac{2 - \sqrt{8}}{2} = \frac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}$ but this is not possible as this is a negative value

$x_2 = \frac{2 + \sqrt{8}}{2} = 1 + \sqrt{2}$

So two interval solutions : $x < 0$ and $x > 1 + \sqrt{2}$

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6 Solve $\frac{x}{x+1} < 1$. (*Note:* This looks like a standard problem, but in fact requires some analysis, depending on which method you use.)

$$\frac{x}{x+1} < 1 \iff x(x+1) < (x+1)^2$$

$$\iff x^2 + x < x^2 + 2x + 1$$

$$\iff x - 2x < 1$$

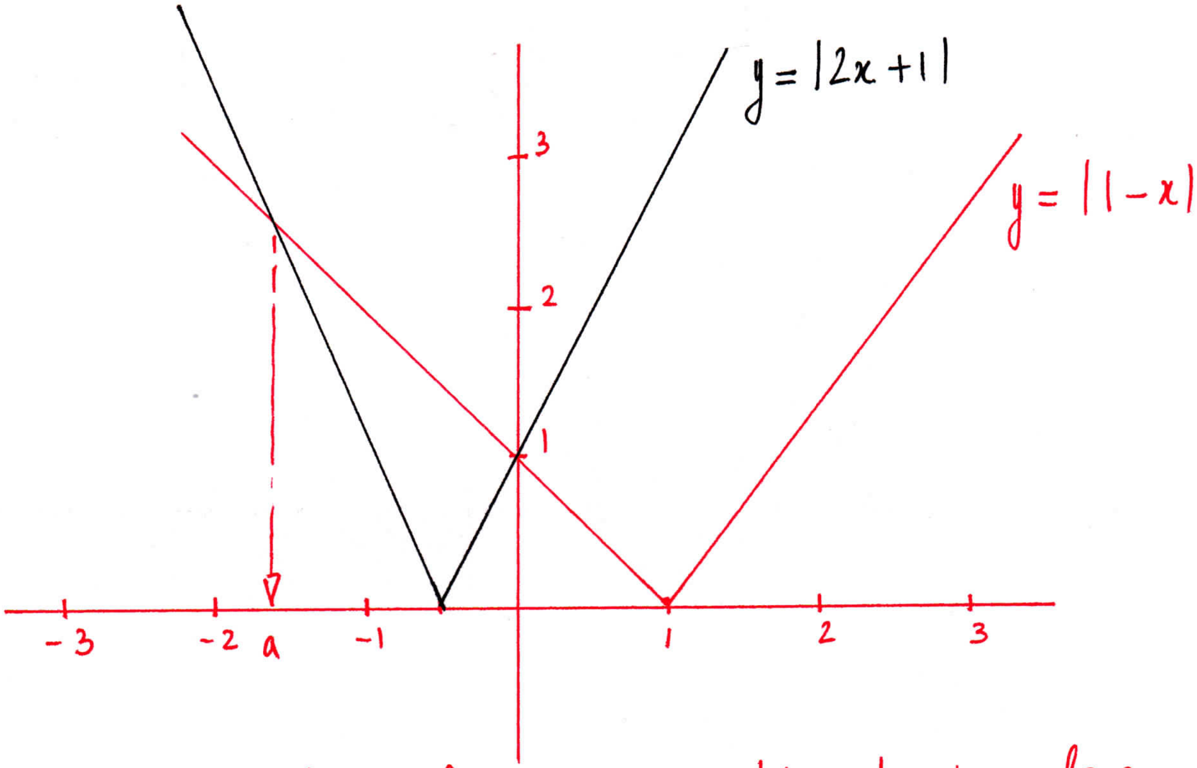
$$\iff -x < 1$$

$$\iff x > -1$$

7 Solve $\left|\frac{1-x}{2x+1}\right| \geq 1$. (Hint: $|2x+1|$ is known to be non-negative.)

$$\left|\frac{1-x}{2x+1}\right| \geqslant 1 \iff \left|\frac{1-x}{2x+1}\right| |2x+1| \geqslant |2x+1|$$

$$\iff |1-x| \geqslant |2x+1|$$



We can see that the curve $y = |1-x|$ is above the curve $y = |2x+1|$ when $a \leqslant x \leqslant 0$

where a is where the lines intercept ~~on the~~ when $x < 0$

i.e. $|1-x| = 1-x$ and $|2x+1| = -2x-1$

i.e. $1-x = -2x-1$

$$\iff 2 = -x$$

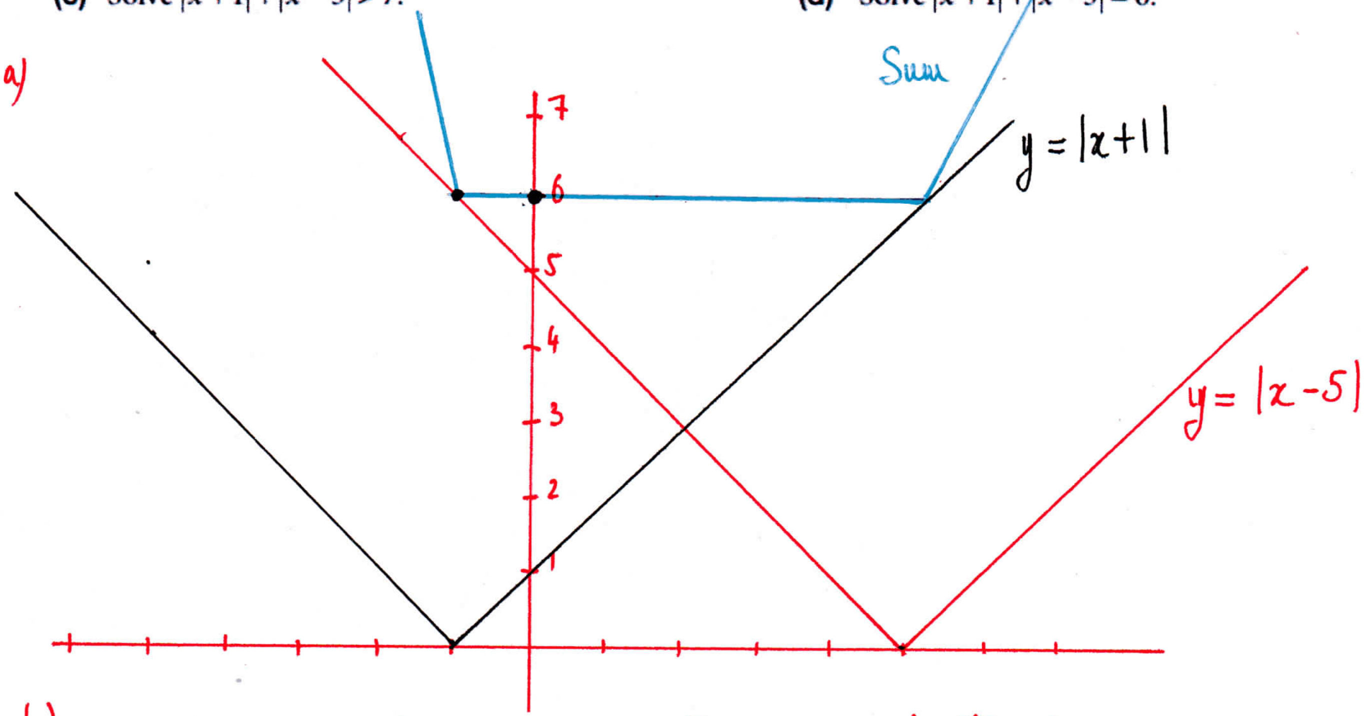
$$x = -2$$ (graph was not very precise ☹)

So the interval solution is $-2 \leqslant x \leqslant 0$

INEQUALITIES INVOLVING ABSOLUTE VALUE AND SQUARE ROOTS

9 **(a)** On the same axes sketch $y=|x+1|$ and $y=|x-5|$. **(b)** Hence graph $y=|x+1|+|x-5|$.
(c) Solve $|x+1|+|x-5|>7$. **(d)** Solve $|x+1|+|x-5|=6$.

a)



b) See graph in blue which is the sum of the 2 curves.

c) This situation occurs when the blue curve is above 7, when

* $x>5$ ie $|x+1|+|x-5| = 2x-4$ i.e $2x-4>7$ $x>11/2$

or when $x<-1$ $|x+1|+|x-5| = -x-1-x+5 = -2x+4$

i.e. $-2x+4>7$ $-2x>3$ $-x>\frac{3}{2}$ $x<-\frac{3}{2}$

So two interval solutions: $x<-\frac{3}{2}$ and $x>11/2$

d) $|x+1|+|x-5|=6$

When $-1<x<5$ then $|x+1| = x+1$ and $|x-5| = -x+5$

So $|x+1|+|x-5| = x+1-x+5=6$

The interval solution is $-1<x<5$. For all these values we have indeed $|x+1|+|x-5|=6$