PROBLEMS INVOLVING QUADRATIC EQUATIONS

Example 17

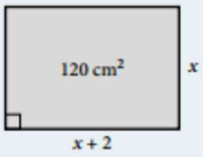
One side of a rectangle is 2 cm longer than the other side. The area of the rectangle is 120 cm^2. What are the dimensions of the rectangle?

Solution

Let one side length be x cm.

The other side length is $(x + 2)$ cm.

Draw a diagram to show this information.



The area of the rectangle is 120 cm^2, so:

$$x(x + 2) = 120$$
$$x^2 + 2x = 120$$
$$x^2 + 2x - 120 = 0$$
$$(x + 12)(x - 10) = 0$$
$$x + 12 = 0 \quad \text{or} \quad x - 10 = 0$$
$$x = -12 \quad \text{or} \quad x = 10$$

Because x represents the side length of a rectangle, $x > 0$. This means the only possible solution is $x = 10$.

Therefore the dimensions of the rectangle are 10 cm by 12 cm.

We should have written the original equation as $x(x + 2) = 120$, $x > 0$, to remember that x must represent a positive length.

Also, don't forget that when solving $AB = 0$, either $A = 0$ or $B = 0$ or $A = B = 0$, but zero is not always a valid solution to the problem. This is one reason why we don't always use all solutions to the quadratic equation in practical problems.

Example 18

The height h metres of a stone, t seconds after being thrown straight up, is given by the equation $h = 30t - 5t^2$. When is the stone at a height of 40 metres?

Solution

$$h = 30t - 5t^2$$
For $h = 40$: $40 = 30t - 5t^2$
$$5t^2 - 30t + 40 = 0$$
$$5(t^2 - 6t + 8) = 0$$
$$5(t - 2)(t - 4) = 0$$
$$t = 2 \quad \text{or} \quad t = 4$$

On the way up, the stone reaches a height of 40 m after 2 seconds; on the way down, it comes back to a height of 40 m at 4 seconds. In this problem both answers make sense.