METHODS OF PROOF

1 Use a direct proof to prove each of the following statements.

(a) The sum of any two odd integers is even.
(b) The sum of an odd integer and an even integer is always odd.
(c) The product of two odd integers is odd.
(d) The sum of two consecutive odd numbers is divisible by 4.
(e) The sum of the squares of five consecutive integers is divisible by 5.
(f) The product of two rational numbers is rational.
(g) The sum of two rational numbers is rational.
(h) If n is odd, then n^2 is odd.

2 Use a contrapositive proof to prove each of the following statements.

(a) Let n be an integer. If $3n + 2$ is even, then n is even.

(b) If a and b are integers and ab is even, then at least one of a and b is even.

(c) Let n be an integer. If $n^3 + 5$ is odd, then n is even.

(d) If x is irrational, then $\sqrt{x}$ is irrational.

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3 Use a proof by contradiction to prove each of the following statements.

(a) $\sqrt{3}$ is irrational.

(c) The sum of a rational and an irrational number is irrational.

(d) The product of a rational and an irrational number is irrational.

4 Prove each of the following logical equivalences.

(a) Let n be a positive integer. $n + 9$ is even if and only if $n + 6$ is odd.

(b) Let n be a positive integer. $n - 3$ is odd if and only if $n + 2$ is even.

(c) Let n be a positive integer. n is even if and only if $13n + 4$ is even.

5 Consider the following statement:

'If two integers have an even product, then at least one of the two integers must be even.'

To prove this statement by contraposition, it would be necessary to:

A suppose that at least one of the two integers is even, and then show that the product must be even.
B suppose that at least one of the integers is odd, and then show that the product must be odd.
C suppose that both integers are odd, and then show that the product must be odd.
D suppose that the two integers have an even product and that both integers are odd, and then show that a contradiction arises.

7 Let a, b, c be positive real numbers such that $ab = c$. Prove that $a \leq \sqrt{c}$ or $b \leq \sqrt{c}$.

9 Prove that a four-digit number is divisible by 9 if and only if the sum of its digits is divisible by 9.

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11 Prove that every odd integer can be expressed as the difference between two perfect squares.

12 Prove by contradiction that if a, b are integers, then $a^2 - 4b - 3 \neq 0$

13 Let k be a positive integer. Prove that if $2^{k+2} + 3^{3k}$ is divisible by 5, then $2^{k+3} + 3^{3k+3}$ is also divisible by 5.

15 Use a proof by contradiction to show that there is no rational solution to the equation $x^3 + x + 1 = 0$. As a hint, start by supposing, for a contradiction that $r = \frac{p}{q}$ is a rational solution to the equation, where p, q are integers with no common factor other than 1 and with $q \neq 0$. Then consider what would happen if both p and q were odd, or if one of them was even and the other odd.