TRIGONOMETRIC EQUATIONS - CHAPTER REVIEW

1 **(a)** Express $2\sqrt{3}\cos\left(\theta+\frac{\pi}{6}\right)-2\cos\theta$ in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$.

(b) Hence, or otherwise, solve $2\sqrt{3}\cos\left(\theta+\frac{\pi}{6}\right)-2\cos\theta=1$ for $0<\theta<2\pi$.

2 **(a)** Express $3\sin x + 4\cos x$ in the form $r\sin(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$.

(b) Hence, or otherwise, solve $3\sin x + 4\cos x = 5$ for $0 \le x \le 2\pi$. Give answer(s) to two decimal places.

(c) Write the general solution for $3\sin x + 4\cos x = 5$.

3 Use $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$ to find the exact value of $\tan\dfrac{\pi}{8}$.

4. First show that $cos\ 3\theta = 4\ cos^3\theta - 3\ cos\ \theta$ [by expanding $cos\ 3\theta$ as $cos\ (2\theta + \theta)$]

Show that the cubic equation $8x^3 - 6x + 1 = 0$ can be reduced to the form $cos\ 3\theta = \frac{-1}{2}$ by substituting $x = cos\ \theta$, and then solve this equation.

From this, deduce the following:

a) $cos\left(\frac{2\pi}{9}\right) + cos\left(\frac{4\pi}{9}\right) + cos\left(\frac{8\pi}{9}\right) = 0$

b) $sec\left(\frac{2\pi}{9}\right) sec\left(\frac{4\pi}{9}\right) sec\left(\frac{8\pi}{9}\right) = -8$

c) $sec\left(\frac{2\pi}{9}\right) + sec\left(\frac{4\pi}{9}\right) + sec\left(\frac{8\pi}{9}\right) = 6$

d) $tan^2\left(\frac{2\pi}{9}\right) + tan^2\left(\frac{4\pi}{9}\right) + tan^2\left(\frac{8\pi}{9}\right) = 33$

5 It can be shown that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. Use this result to solve $\cos 3\theta + \cos 2\theta + \cos\theta = 0$ for $0 \leq \theta \leq 2\pi$.

6 **(a)** Expand $\cos(2A + B)$ and hence prove that $\frac{1}{4}\cos 3\theta = \cos^3\theta - \frac{3}{4}\cos\theta$.

(b) By writing $x = k\cos\theta$ and giving k a suitable value, use the formula proved in part **(a)** to find the three roots of the equation $27x^3 - 9x = 1$. Hence write the value of the product $\cos\frac{\pi}{9}\cos\frac{3\pi}{9}\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}$.

7 If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation $x^3 - (a+1)x^2 + (c-a)x - c = 0$, show that $\alpha + \beta + \gamma = n\pi + \frac{\pi}{4}$.

8 Solve $\sin x = \cos 5x$ for $0 < x < \pi$.

9 (a) Find A and B in terms of x and y such that $\sin x + \sin y = 2\sin A \cos B$.

(b) Find the solution of $\sin\theta + \sin 2\theta + \sin 3\theta = 0$ for $0 \leq \theta \leq \pi$.