FUNDAMENTAL COUNTING PRINCIPLE

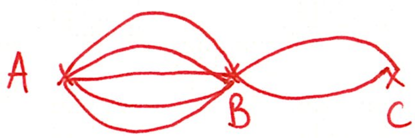
Question 1: Use the definition of $n!$ to estimate:

$4! = 4\times3\times2\times1$ $= 24$	$\frac{15!}{14!} = 15$	$\frac{9!}{4!} = 9\times8\times7\times6\times5$ $= 15,120$	$\frac{12!}{3!\times9!} = 220$ (use calculator)

Question 2: Simplify:

$\frac{n!}{(n-1)!} = n$	$\frac{(n-2)!}{n!} = \frac{(n-2)!}{n(n-1)(n-2)!}$ $= \frac{1}{n(n-1)}$	$\frac{(n+1)!}{(n-1)!} = (n+1)(n)$ $= n(n+1)$
$\frac{1}{n!} + \frac{1}{(n+1)!} =$ $= \frac{n+1}{(n+1)!} + \frac{1}{(n+1)!}$ $= \frac{n+1+1}{(n+1)!}$ $= \frac{n+2}{(n+1)!}$	$(n+1)! + (n-1)! =$ $= (n-1)!\,[(n+1)\times n + 1]$ $= [n^2+n+1]\,(n-1)!$	$9! + 8! + 7! =$ $= 7!\,[8\times9 + 8 + 1]$ $= 81 \times 7!$

1 There are five roads from town A to town B, and two roads from town B to town C. In how many different ways can you travel by road from A to B to C?



$5 \times 2 = 10$ different ways to travel from A to C

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2 A man has three pairs of shoes, four suits and six ties. How many different sets of shoes, suits and ties can he wear?

$3 \times 4 \times 6 = 72$ possible outfits

3 In how many ways can seven books be arranged in a row?

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = 5{,}040$ ways.

4 A Mathematics test contains 20 multiple-choice questions. Each question has four possible answers, *A*, *B*, *C* and *D*. If a student guesses every answer, in how many different ways can the answers be given?

4^{20} different ways.

5 A restaurant menu has three choices of soup, five choices of main course and three choices of dessert. How many different meals of soup, main course and dessert are possible?

$3 \times 5 \times 3 = 45$ possible ways.

6 There are 10 candidates for school captain and vice-captain. The number of different ways they might be selected is: **A** 90 **B** 45 **C** 10 **D** 9

10 choices for captain, then 9 choices for vice-captain

So $10 \times 9 = 90$ possible ways.

Response $\boxed{A}$

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7 New South Wales black-and-white number plates consist of three letters, two digits and one letter. How many different number plates can be made?

$$26^3 \times 10^2 \times 26 = 26^4 \times 100 = 45,697,600$$

possible ways for number plates.

8 How many arrangements of the letters of the word PENCIL are possible? All letters are different

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$ possible ways.

9 In how many different ways can *A*, *B*, *C*, *D*, *E* be arranged:

(a) in a row of three **(b)** in a row all together?

a) $5 \times 4 \times 3 = 60$ possible different ways.

b) $5! = 120$ possible different ways.

10 The newer New South Wales black-and-yellow number plates consist of two letters, two digits and two letters.

(a) How many different number plates can be made?

(b) What is the reason for changing the number plates from three letters and three digits to two letters, two digits, two letters?

a) $26^2 \times 10^2 \times 26^2 = 100 \times 26^4 = 45,697,600$ different ways

b) $26^3 \times 10^3 = 1000 \times 26^3 = 17,576,000$ different ways

which is about 3 times less than the number at a)

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11 The Olympic teams from eight countries are swimming in the 200-metre relay final. How many different finishing results are possible? (Assume no tied results.)

$8! = 40{,}320$ possible different finishing results.

12 The digits 0 to 9 are used to make 10-digit numbers (not beginning with zero). How many different numbers are possible if:

(a) each digit can be used only once **(b)** each digit can be used any number of times?

a) 9 choices for the 1st digit

then $9!$ for subsequents digits.

So total is $9 \times 9! = 3{,}265{,}920$ possible different numbers

b) For the 1st digit, 9 choices.

then 10^9 for subsequent digits. (10 possible choices for each digit)

Total is $9 \times 10^9 = 9{,}000{,}000{,}000$ (9 billion)