ARITHMETIC OF COMPLEX NUMBERS

1 $i^5 = \ldots$

A 1 **B** -1 **C** i **D** $-i$

2 Solve the following equations.

(a) $z^2 + 9 = 0$ **(b)** $z^2 + 25 = 0$ **(c)** $z^2 + 2z + 17 = 0$

3 Simplify:

(a) i^3 **(b)** i^4 **(c)** i^6 **(d)** i^7 **(e)** i^8

4 If $z = 5 - 2i$, find:

a)	$\overline{z}$
b)	$z\overline{z}$
c)	z^2
d)	$(z - \overline{z})^2$
e)	$\dfrac{z-1}{z-i}$

f) z^{-1}

5 Simplify:

a) $(3 + 5i) + (7 - 2i) =$
b) $(4 + 7i) - (-2 + 9i) =$
c) $(5 + 2i)(3 - 4i) =$
d) $(7 - 3i)(7 + 3i) =$
e) $(2 - 5i)^2 =$
f) $i^{17} =$
g) $(\sqrt{3} + 2i)(\sqrt{3} - 2i) =$
h) $\frac{1}{2+3i} =$

i) $\frac{8+5i}{4-3i} =$
j) $\frac{3i}{2+5i} + \frac{2}{2-5i} =$

6 Find real numbers x and y such that:

(a) $(x + iy)(2 - 3i) = -13i$ **(b)** $(1 + i)x + (2 - 3i)y = 10$

7 If $z_1 = 3 + i$ and $z_2 = 2 - 3i$, find:

(a) $(z_1 - z_2)^2$
(b) $\overline{z_1} \times \overline{z_2}$
(c) $\overline{z_1 z_2}$
(d) $\dfrac{z_1 - z_2}{z_1 + z_2}$

8 Find the linear factors of the following expressions.

(a) $z^2 + 9$ **(b)** $z^2 + 36$ **(c)** $(z - 3)^2 + 16$ **(d)** $(2z + 3)^2 + 8$

(e) $z^2 + 2z + 26$ **(f)** $z^2 - 6z + 20$ **(g)** $2z^2 + 2z + 4$ **(h)** $z^3 + 1000$

9 Solve the equation: **(a)** $2z - 1 = (4 - i)^2$ **(b)** $\frac{z - 2}{z} = 1 + i$

12 **(a)** Show that $\sqrt{3} - i$ is a root of the equation $z^3 - (\sqrt{3} - i)z^2 + 9z - 9\sqrt{3} + 9i = 0$.

(b) Find the other two solutions of the equation.

(c) Use your answer to part **(b)** to verify that the results for the sum of roots, for the sum of products of pairs of roots and for the product of roots of a cubic equation are true when the coefficients and roots are complex numbers.

13 Solve the following quadratic equations.

(a) $z^2 - (3 - 2i)z + (1 - 3i) = 0$

(b) $z^2 - z + (4 + 2i) = 0$

14 Let $z = a + ib$ where a, b are real. Prove that there are always two square roots of z except when $a = b = 0$.

15 If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, show that the following equations are true.

(a) $z_1 + \overline{z_1} = 2 \times \text{Re}(z_1)$	**(b)** $z_1 - \overline{z_1} = 2 \times \text{Im}(z_1) \times i$
(c) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$	**(d)** $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$
(e) $\overline{z_1 \times z_2} = \overline{z_1} \times \overline{z_2}$	