AREA BETWEEN TWO CURVES

1 Calculate the area of the region bounded by the line $y = 2x$ and the parabola $y = x^2$.

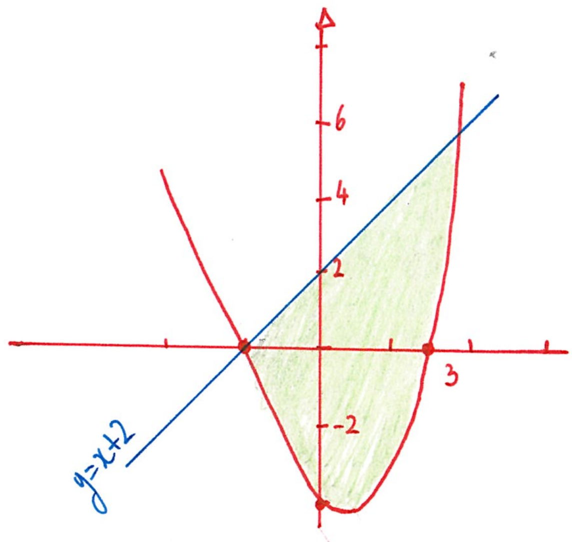
The two curves cross when $2x = x^2$, or $x^2 - 2x = 0$

$\Leftrightarrow x(x-2) = 0$, so at $x = 0$ and at $x = 2$.

Both curves are above the x-axis, and on $[0,2]$ $2x \geqslant x^2$

The area between the curves is $\int_0^2 (2x - x^2)\,dx$

$$\int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = 2^2 - \frac{2^3}{3} = 4 - \frac{8}{3} = \frac{4}{3} \text{ units}^2$$



3 The area of the region bounded by the line $y = x + 2$ and the parabola $y = x^2 - 4$ is given by:

$\boxed{\text{A}}$ $\int_{-2}^{3}(6+x-x^2)\,dx$ B $\int_{-3}^{2}(6+x-x^2)\,dx$ C $\int_{-2}^{3}(x^2-x-6)\,dx$ D $\int_{-3}^{2}(x^2-x-6)\,dx$

The two curves cross when $x + 2 = x^2 - 4 \Leftrightarrow x^2 - x - 6 = 0$

$\Leftrightarrow (x+2)(x-3) = 0$ so at $x = -2$ and $x = +3$.

On this interval, $x + 2 \geqslant x^2 - 4$. So the area is represented by

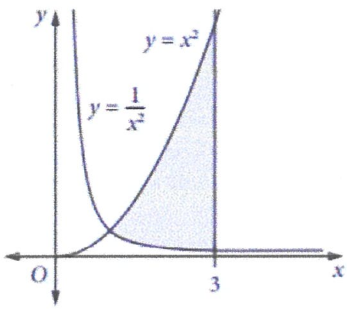
$$\int_{-2}^{3} (2+x) - (x^2 - 4)\,dx$$

or $$\int_{-2}^{3} (-x^2 + x + 6)\,dx$$

Response $\boxed{\text{A}}$

AREA BETWEEN TWO CURVES

4 Calculate the area bounded by $f(x) = x^2$, $g(x) = \frac{1}{x^2}$, $x > 0$, the x-axis and the line $x = 3$.



The two lines cross when $x^2 = \frac{1}{x^2}$

i.e. $x^4 = 1$ or $x = 1$

So we need to calculate $\int_1^3 \left(x^2 - \frac{1}{x^2}\right) dx$

$$\int_1^3 \left(x^2 - \frac{1}{x^2}\right) dx = \left[\frac{x^3}{3} - \frac{x^{-2+1}}{(-2+1)}\right]_1^3$$

$$= \left[\frac{x^3}{3} - \frac{x^{-1}}{-1}\right]_1^3$$

$$= \left[\frac{x^3}{3} + \frac{1}{x}\right]_1^3$$

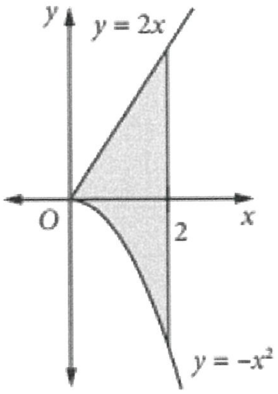
$$= \left[\frac{3^3}{3} + \frac{1}{3}\right] - \left[\frac{1^3}{3} + \frac{1}{1}\right]$$

$$= \left(9 + \frac{1}{3}\right) - \left(\frac{1}{3} + 1\right)$$

$$= 8 \text{ units}^2$$

AREA BETWEEN TWO CURVES

7 Find the area enclosed by the line $y = 2x$, the parabola $y = -x^2$ and the line $x = 2$.



$$\text{Area} = \int_0^2 \left[2x - (-x^2)\right] dx$$

$$\text{Area} = \int_0^2 (x^2 + 2x)\, dx$$

$$\text{Area} = \left[\frac{x^3}{3} + x^2\right]_0^2$$

$$\text{Area} = \frac{2^3}{3} + 2^2 = \frac{8}{3} + 4 = \frac{20}{3} = 6\,\frac{2}{3} \text{ units}^2$$

9 Calculate the area of the region enclosed by the graphs of the parabola $y = 2x^2 - 5x - 3$ and the line $y = 3x - 3$. Indicate whether each statement below is a correct or incorrect step in calculating this area.

correct **(a)** Intersection points: $(0, -3)$ and $(4, 9)$

(b) Area $= \int_0^4 (8x - 2x^2)\,dx$ correct

~~(c)~~ Area $= \int_{-3}^9 (8x - 2x^2)\,dx$

(d) Area $= 21\frac{1}{3}$ units^2 correct

$$2x^2 - 5x - 3 = 3x - 3$$

$$\Leftrightarrow 2x^2 - 8x = 0$$

$$\Leftrightarrow x^2 - 4x = 0$$

$$\Leftrightarrow x(x-4) = 0$$

either $x = 0$ or $x = 4$

$y = -3$ $\quad$ $y = 9$

$$\text{Area} = \int_0^4 (3x-3) - (2x^2 - 5x - 3)\, dx$$

$$\text{Area} = \int_0^4 (-2x^2 + 8x)\, dx$$

$$\text{Area} = \left[-\frac{2x^3}{3} + \frac{8x^2}{2}\right]_0^4$$

$$\text{Area} = \left[-\frac{2\ 4^3}{3} + 4 \times 4^2\right]$$

$$\text{Area} = \frac{64}{3} = 21\,\frac{1}{3} \text{ units}^2$$

AREA BETWEEN TWO CURVES

13 A straight line through the origin cuts the parabola $y = 4x - x^2$ at the point where $x = 3$.

(a) Find the equation of this line.

(b) Calculate the area of the region bounded by the parabola and the straight line.

~~**(c)** Calculate the area of the region bounded by the parabola, the straight line and the x-axis.~~

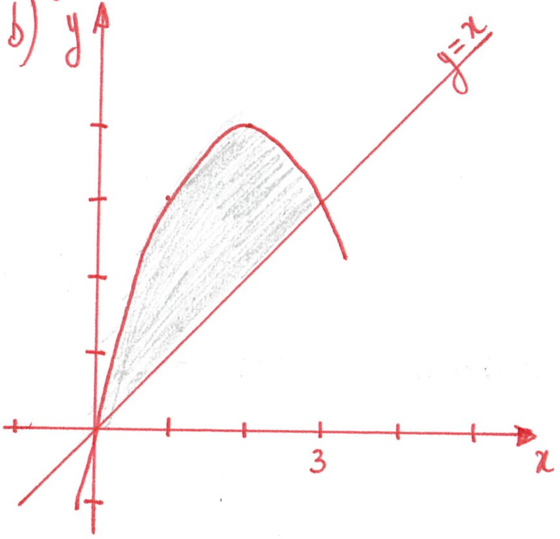
a) The line has for general equation $y = kx$

$kx = 4x - x^2$ when $x = 3$, i.e. $3k = 4 \times 3 - 3^2 = 3$

so $k = 1$

$y = x$

b)



$\int_0^3 \left[(4x - x^2) - x\right] dx$ is the area

$$\int_0^3 \left[(4x - x^2) - x\right] dx = \int_0^3 \left[-x^2 + 3x\right] dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3$$

$$= -\frac{27}{3} + \frac{3 \times 9}{2}$$

$$= -9 + \frac{3}{2} \times 9$$

$$= \frac{9}{2} = 4.5 \text{ units}^2$$