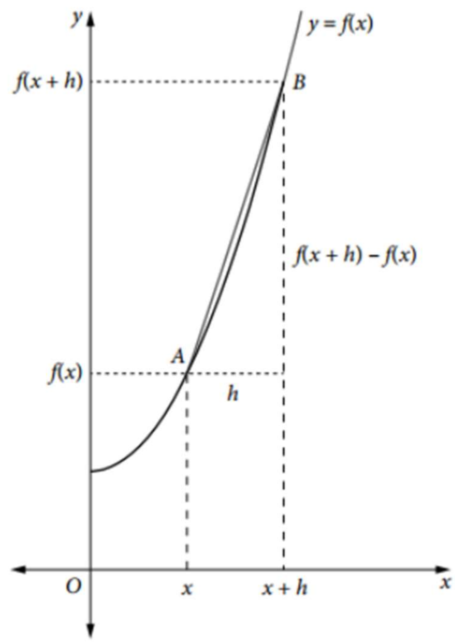
FIRST PRINCIPLE OF DIFFERENTIATION

On the graph below, we draw the secant to the graph between two points of this graph, i.e. the point $(x, f(x))$ and another point a bit further away, at a distance h from the first point, so its coordinates are $(x + h, f(x + h))$



The ratio $\frac{f(x+h)-f(x)}{(x+h)-x}$, which simplifies as $\frac{f(x+h)-f(x)}{h}$, represents the gradient of the line joining the points $(x, f(x))$ and $(x + h, f(x + h))$.

When h becomes smaller and smaller (i.e. tends towards 0), the value of the ratio $\frac{f(x+h)-f(x)}{h}$ tends towards the gradient of the tangent to the curve. This value is called "the derivative of the function f at x", and is noted either $f'(x)$ ("Lagrange notation"; Lagrange was a French mathematician) or $\frac{df}{dx}$ ("Leibniz notation"; Leibniz was a German mathematician)

So: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ this calculation is called "first principle of differentiation"

Example of such calculation for $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2hx - h^2 - x^2}{h} = \lim_{h \to 0} \frac{2hx - h^2}{h} = \lim_{h \to 0}(2x - h) = 2x$$

So when $f(x) = x^2$, then $f'(x) = 2x$ (can also be noted: $\frac{df(x)}{dx} = 2x$)

What it means is that the gradient of the curve $f(x) = x^2$ at a point $x = c$ is $f'(c) = 2c$

For example, the gradient of the curve $f(x) = x^2$ at $x = 1$ is $f'(1) = 2 \times 1 = 2$

DERIVATIVES OF THE FUNCTIONS $\boldsymbol{f(x) = c}$, $\boldsymbol{f(x) = x}$ AND $\boldsymbol{f(x) = x^2}$

Derivative of a constant function $\boldsymbol{f(x) = c}$

If $f(x) = c$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = 0$

Therefore if $f(x) = c$ then $f'(x) = 0$

Example: if $f(x) = 3$ then $f'(x) = 0$

Derivative of the function $\boldsymbol{f(x) = x}$

If $f(x) = x$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h) - x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$

Therefore if $f(x) = x$ then $f'(x) = 1$

Derivative of the function $\boldsymbol{f(x) = x^2}$

If $f(x) = x^2$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$$f'(x) = \lim_{h \to 0} \left(\frac{x^2 + 2xh + h^2 - x^2}{h} \right)$$

$$f'(x) = \lim_{h \to 0} \left(\frac{2xh + h^2}{h} \right)$$

$$f'(x) = \lim_{h \to 0} (2x + h)$$

Therefore if $f(x) = x^2$ then $f'(x) = 2x$