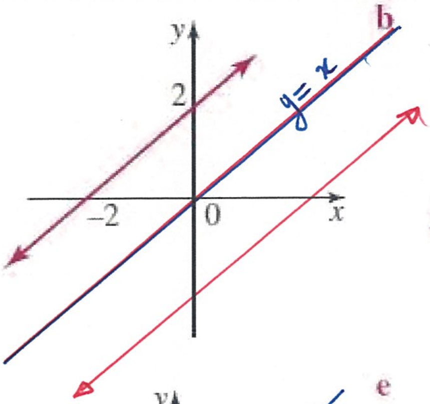
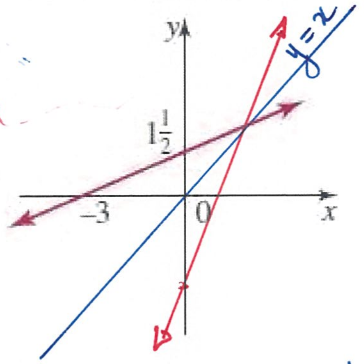
INVERSE FUNCTIONS

1 Sketch the inverse relation for each of the following.

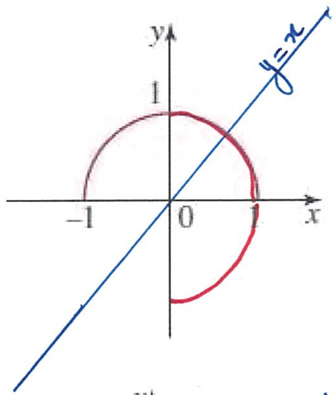
a



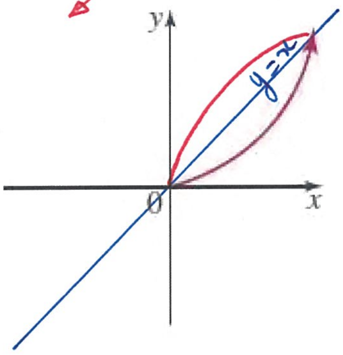
b



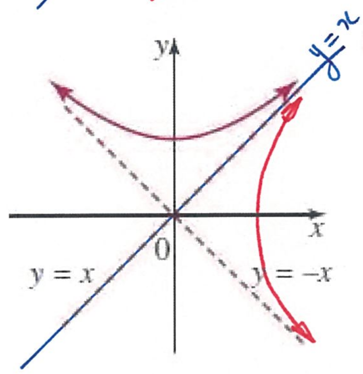
c



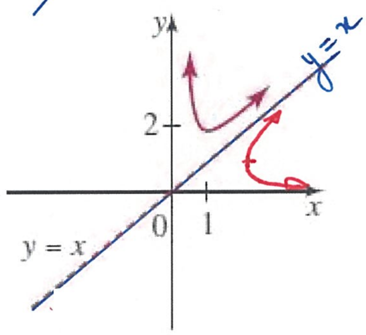
d



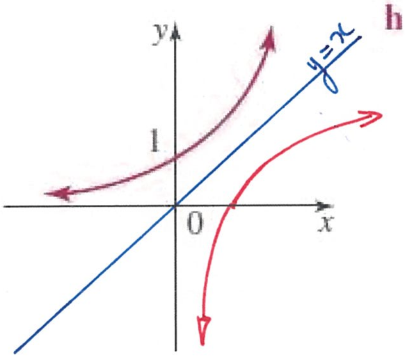
e



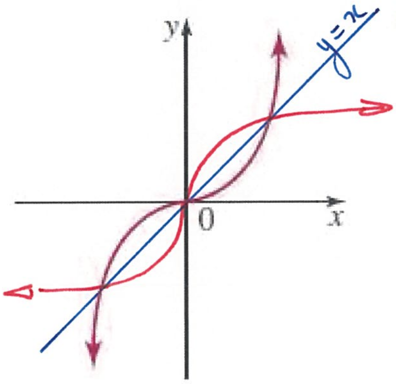
f



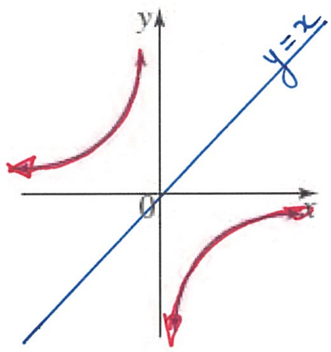
g



h



i



2 Which of the relations in question 1 have an inverse that is a function?

a) b) d) g) h) i)

NOT a function → c), e) f)

INVERSE FUNCTIONS

10. SHow that the functions $f(x)$ and $g(x)$ are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$

a) $f(x) = x + 7$ and $g(x) = x - 7$ $f(g(x)) = (x-7)+7 = x$ $g(f(x)) = (x+7) - 7 = x$	b) $f(x) = 5x$ and $g(x) = \frac{x}{5}$ $f(g(x)) = 5(x/5) = x$ $g(f(x)) = \frac{(5x)}{5} = x$
c) $f(x) = 2x + 2$ and $g(x) = \frac{x}{2} - 1$ $f(g(x)) = 2\left(\frac{x}{2} - 1\right) + 2 = x - 2 + 2 = x$ $g(f(x)) = \frac{(2x+2)}{2} - 1 = x + 1 - 1 = x$	d) $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ $f(g(x)) = \left[\sqrt[3]{x-1}\right]^3 + 1 = x - 1 + 1 = x$ $g(f(x)) = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$
e) $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{1}{x} - 3$ $f(g(x)) = \frac{1}{\left(\frac{1}{x} - 3\right) + 3} = \frac{1}{\frac{1}{x}} = x$ $g(f(x)) = \frac{1}{\left(\frac{1}{x+3}\right)} - 3 = x + 3 - 3 = x$	f) $f(x) = \frac{x-1}{x+2}$ and $g(x) = \frac{2x+1}{1-x}$ $f(g(x)) = \frac{\left(\frac{2x+1}{1-x}\right) - 1}{\left(\frac{2x+1}{1-x}\right) + 2} = \frac{2x+1-(1-x)}{2x+1+2(1-x)} = \frac{3x}{3} = x$ $g(f(x)) = \frac{2\left(\frac{x-1}{x+2}\right) + 1}{1 - \left(\frac{x-1}{x+2}\right)} = \frac{2(x-1)+x+2}{x+2-(x-1)} = \frac{3x}{3} = x$

4 Find the inverse function for each of the following functions. For each inverse, make y the subject.

a $y = \frac{1}{x} - 2$ **b** $y = \frac{1}{x-1}$ **c** $y = \frac{x-3}{x+3}$ **d** $y = \frac{2x}{5-x}$

a) $\Leftrightarrow y + 2 = \frac{1}{x}$ so $x = \frac{1}{y+2}$ $\therefore$ the inverse function is $f^{-1}(x) = \frac{1}{x+2}$

b) $x - 1 = \frac{1}{y}$ so $x = \frac{1}{y} + 1$ $\therefore$ the inverse function is $f^{-1}(x) = \frac{1}{x} + 1$

c) $(x+3)y = x - 3$ so $x(y-1) = -3 - 3y \Leftrightarrow x = -3\frac{(y+1)}{y-1} = \frac{3(y+1)}{1-y}$

so $f^{-1}(x) = \frac{3(x+1)}{1-x}$

d) $(5-x)y = 2x \Leftrightarrow -xy - 2x = -5y \Leftrightarrow x(y+2) = 5y \Leftrightarrow x = \frac{5y}{y+2}$

so $f^{-1}(x) = \frac{5x}{x+2}$

INVERSE FUNCTIONS

3 For each of the following, find the inverse function and state the domain and range of the inverse.

(a) $f(x) = 2x - 4$ **(b)** $f(x) = x^2 - 1, x \geq 0$ **(c)** $g(x) = \sqrt{x-3}$ **(d)** $f(x) = \sqrt{9-x^2}, -3 \leq x \leq 0$

$y = 2x - 4$

a) $2x = y + 4 \iff x = \frac{y+4}{2}$ so $f^{-1}(x) = \frac{x+4}{2} = \frac{x}{2} + 2$

Domain of f^{-1} is $\mathbb{R}$ Range of f^{-1} is $\mathbb{R}$

b) $y = x^2 - 1$ so $x^2 = y + 1 \iff x = \sqrt{y+1}$

so $f^{-1}(x) = \sqrt{x+1}$ Domain of f^{-1} is $[-1, +\infty)$

Range of f^{-1} is $\mathbb{R}^+$

c) $y = \sqrt{x-3}$ so $y^2 = x - 3$ so $x = y^2 + 3$

$f^{-1}(x) = x^2 + 3$ Domain is $\mathbb{R}$

Range is $[3, +\infty)$

d) $y = \sqrt{9 - x^2}$ so $y^2 = 9 - x^2 \iff x^2 = 9 - y^2$

so $x = \pm\sqrt{9 - y^2}$ But $x < 0$ as noted above,

so $x = -\sqrt{9-y^2}$ $f^{-1}(x) = -\sqrt{9-x^2}$

Domain is $[-3, 3]$

Range is $[-3, 0]$

INVERSE FUNCTIONS

6 Show that the following pairs of functions are inverses by showing that $f(g(x)) = g(f(x)) = x$.

(d) $f(x) = 2x - x^2, x \geq 1$ and $g(x) = 1 + \sqrt{1-x}, x \leq 1$ **(e)** $f(x) = \frac{1}{2x-1}, x > \frac{1}{2}$ and $g(x) = \frac{x+1}{2x}, x > 0$

d) $f[g(x)] = 2(1+\sqrt{1-x}) - (1+\sqrt{1-x})^2$

$f[g(x)] = 2 + 2\sqrt{1-x} - [1 + 2\sqrt{1-x} + (1-x)]$

$f[g(x)] = \cancel{2} + \cancel{2\sqrt{1-x}} - \cancel{2} - \cancel{2\sqrt{1-x}} + x = x$

$g[f(x)] = 1 + \sqrt{1-(2x-x^2)} = 1 + \sqrt{x^2 - 2x + 1}$

$g[f(x)] = 1 + \sqrt{(x-1)^2} = 1 + (x-1) = x$

e) $f[g(x)] = \dfrac{1}{2\left(\frac{x+1}{2x}\right) - 1} = \dfrac{1}{\frac{x+1}{x} - 1} = \dfrac{x}{x+1-x} = \dfrac{x}{1} = x$

$g[f(x)] = \dfrac{\left(\frac{1}{2x-1}\right) + 1}{2\left(\frac{1}{2x-1}\right)} = \dfrac{1 + (2x-1)}{2} = \dfrac{2x}{2} = x$