EXPECTED VALUE, VARIANCE AND STANDARD DEVIATION OF DISCRETE PROBABILITY DISTRIBUTIONS

1 Find $E(X)$ for each of the following probability distributions.

(a)

x	1	3	5	7	9
$P(X=x)$	0.2	0.3	0.25	0.15	0.1

(b)

x	−1	0	1	2	3
$P(X=x)$	0.4	0.15	0.2	0.05	0.2

2 Use the given value of $E(X)$ to solve for the unknowns.

(a) Find the values of a and b in the following probability distribution, given that $E(X) = 3.8$.

x	2	3	4	5	6
$P(X=x)$	0.4	0.1	a	0.1	b

3 For the following probability distribution, find:

x	0	1	2	3	4
$P(X=x)$	0.25	0.3	0.2	0.15	0.1

(a) $E(X)$ **(b)** $E(2X-3)$

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6 For the following probability distribution, find:

y	-3	-2	-1	0	1	2
$P(Y=y)$	0.02	0.03	0.25	0.35	0.3	0.05

(a) the standard deviation of Y **(b)** $P(\mu - 2\sigma \leq Y \leq \mu + 2\sigma)$.

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11 Eric just got a new job selling cars. He is offered a choice of two salary packages. In the first package he receives a weekly retainer of $200 and an additional $650 for every car sold. In the second package his retainer would be $400, but he would only receive $400 for every car sold. Past sales patterns indicate that the probability distribution for the number of cars sold per week is as follows:

Number of vehicles	0	1	2	3	4	5
Probability	0.45	0.35	0.1	0.05	0.04	0.01

Which salary package would Eric be better off taking?

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13 A random variable, T, has the following probability distribution:

t	$w-3$	$w-2$	$w-1$	w	$w+1$
$P(T=t)$	0.2	0.5	0.1	0.05	0.15

(a) Given that $E(T) = 8.45$, find the value of w.

(b) Find $\text{Var}(T)$.

(c) Find the standard deviation of T correct to 2 decimal places.

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14 The probability distribution of G is given by:

$$P(G=g)=\begin{cases} k(6-g) & \text{if } g \in \{0,1,2,3,4\} \\ 0 & \text{for all other values of } g \end{cases}$$

Find the following values: **(a)** k **(b)** $E(G)$ **(c)** $\text{Var}(G)$ **(d)** σ

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16 Enrico runs a game of chance at the Sydney Show. The player pays a fee to play, and draws a card at random from a normal pack of 52 playing cards. If the card is black, the player gets \$1 and their fee is refunded. If the card is a diamond, the player gets \$5 and their fee is refunded.

(a) Find Enrico's loss for the following conditions:
  (i) the card the player chooses is black
  (ii) the card the player chooses is a diamond.

(b) Describe the event for which Enrico keeps the game fee.

(c) Let Enrico's profit for the event described in part **(b)** be \$$p$, and draw up a probability distribution table that shows the three possible outcomes from Enrico's point of view.

(d) Find the value of p so that the expected value for the distribution is 0.

(e) If Enrico wants to make a profit, what is the minimum whole dollar amount he should charge to play the game?

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18 A spinner has nine equal sections, of which five are yellow, three are blue and one is red. If the spinner lands on yellow, you receive \$1. If it lands on blue you receive \$3 and if it lands on red you receive \$5. Let X stand for the amount of money you receive.

(a) Draw up a probability distribution table for this game.

(b) What is the expected value of X?

(c) If the game is to be fair, how much should you pay to play?

EXPECTED VALUE, VARIANCE AND STANDARD DEVIATION OF DISCRETE PROBABILITY DISTRIBUTIONS

19 The game of 'Take a Chance' requires the player to roll three dice, of which one is blue, one is red and one is white. If a 1 shows on the blue die the player receives \$1, if a 1 shows on the red die the player receives \$2 and if a 1 shows on the white die the player receives \$5. In all other circumstances the player receives nothing. A player can receive more than one prize.

(a) Find the probability that the player receives the following amounts:
(i) \$1 **(ii)** \$2 **(iii)** \$3 **(iv)** \$5 **(v)** \$6 **(vi)** \$7 **(vii)** \$8 **(viii)** \$0

(b) What is the expected return on this game?

(c) How much should the operator charge to play if the game is fair?