FURTHER ALGEBRAIC TECHNIQUES - CHAPTER REVIEW

1 Solve:

(a) $5a - 6 = 4(2a + 3)$

$\Leftrightarrow 5a - 6 = 8a + 12$

$\Leftrightarrow -3a = 18$

$\Leftrightarrow a = -\frac{18}{3}$

$\Leftrightarrow a = -6$

(b) $3(8a - 2) - 3(2a + 4) = 0$

$\Leftrightarrow 24a - 6 - 6a - 12 = 0$

$\Leftrightarrow 18a - 18 = 0$

$\Leftrightarrow a = 1$

(c) $8(x + 2) - 3(x + 5) = 2(x - 2)$

$8x + 16 - 3x - 15 = 2x - 4$

$\Leftrightarrow 5x + 1 = 2x - 4$

$\Leftrightarrow 3x = -5$

$\Leftrightarrow x = -5/3$

2 Solve:

(a) $\frac{x}{5} = \frac{3}{20}$

$\Leftrightarrow 20x = 15$

$\Leftrightarrow x = \frac{15}{20}$

$\Leftrightarrow x = \frac{3}{4}$

(b) $\frac{3x - 1}{5} = \frac{x}{20}$

$20(3x - 1) = 5x$

$\Leftrightarrow 60x - 20 = 5x$

$\Leftrightarrow 55x = 20$

$\Leftrightarrow x = \frac{20}{55}$

$\Leftrightarrow x = \frac{4}{11}$

(c) $\frac{x - 2}{x + 3} = \frac{3}{5}$

$\Leftrightarrow 5(x - 2) = 3(x + 3)$

$\Leftrightarrow 5x - 10 = 3x + 9$

$\Leftrightarrow 2x = 19$

$\Leftrightarrow x = \frac{19}{2}$

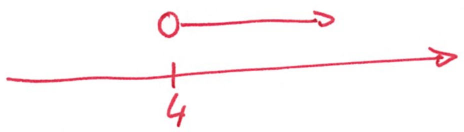
3 Solve, showing your solution on a number line:

(a) $\frac{3x-2}{5} > 2$ (b) $-8 \le 3x - 2 < 16$ (c) $|x-1| > 1$

$\iff 3x - 2 > 10$

$\iff 3x > 12$

$\iff x > 4$



b) $\iff -6 \leqslant 3x \leqslant 18$

$\iff -2 \leqslant x \leqslant 6$

c) $|x-1| > 1$

1) if $x - 1 > 0$ (i.e. $x > 1$)

then $|x-1| = x - 1$, and the inequality is rewritten $x - 1 > 1 \iff x > 2$

So we must have at the same time $x > 1$ and $x > 2$, so $x > 2$

2) if $x - 1 < 0$ (i.e. $x < 1$) then $|x-1| = -x + 1$ and the inequality can be rewritten $-x + 1 > 1 \iff -x > 0$

$\iff x < 0$

So we must have at the same time $x < 1$ and $x < 0$

$\therefore x < 0$

FURTHER ALGEBRAIC TECHNIQUES - CHAPTER REVIEW

4 Solve:

(a) $x^2 = 4$

a) $x = \pm\sqrt{4}$

so $x = \pm 2$

(b) $x^2 = 4x$

$\Leftrightarrow x^2 - 4x = 0$

$\Leftrightarrow x(x-4) = 0$

so $x = 0$ or

$x = 4$

(c) $x^2 = 4x - 4$

$\Leftrightarrow x^2 - 4x + 4 = 0$

$\Leftrightarrow (x-2)^2 = 0$

so $x = 2$

(d) $(x^2 - 3x)^2 = 16$

$\Leftrightarrow x^2 - 3x = \pm\sqrt{16}$

$\Leftrightarrow x^2 - 3x = \pm 4$

1) if $x^2 - 3x = 4$

$\Leftrightarrow x^2 - 3x - 4 = 0$

$\Leftrightarrow (x+1)(x-4) = 0$

so $x = -1$ or $x = 4$

2) if $x^2 - 3x = -4$

$\Leftrightarrow x^2 - 3x + 4 = 0$

$\Delta < 0 \therefore$ no solutions.

So overall, only 2 solutions

$x = -1$ and $x = 4$

(e) $(x^2 - 3x - 10)(x^2 - 3x - 4) = 0$

either $x^2 - 3x - 10 = 0$

or $x^2 - 3x - 4 = 0$

1) if $x^2 - 3x - 10 = 0$

$\Delta = 9 - 4 \times (-10) = 49$

$\Delta > 0 \therefore$ 2 solutions

$x = \dfrac{3+7}{2} = 5$

or $x = \dfrac{3-7}{2} = -2$

2) if $x^2 - 3x - 4 = 0$

2 solutions $x = -1$

and $x = 4$

Overall, 4 solutions

$x = -2$, $x = -1$,

$x = 4$ and $x = 5$

(f) $6x^2 + 7x - 3 = 0$

$\Delta = 49 - 4 \times (-3) \times 6$

$\Delta = 121 = 11^2$

$\Delta > 0$ so 2 solutions

$x = \dfrac{-7+11}{2} = 2$

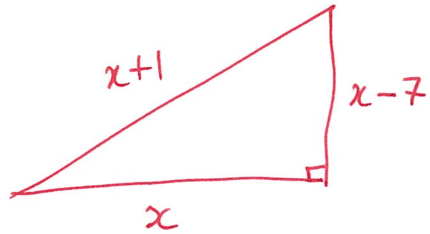
and $x = \dfrac{-7-11}{2} = -9$

So 2 solutions

$x = 2$ and $x = -9$

5 The hypotenuse of a right-angled triangle is $(x + 1)$ cm in length and the other two sides are x cm and $(x - 7)$ cm. Form an equation and solve it to find the length of each side.

$(x+1)^2 = x^2 + (x-7)^2$ (Pythagoras)



$\Leftrightarrow x^2 + 2x + 1 = x^2 + x^2 - 14x + 49$

$\Leftrightarrow -x^2 + 16x - 48 = 0$

$\Leftrightarrow x^2 - 16x + 48 = 0$

$\Delta = 16^2 - 4 \times 48 = 64 = 8^2$ so $x = \dfrac{16-8}{2} = 4$

or $x = \dfrac{16+8}{2} = 12$

$x = 4$ is not possible as $(x-7)$ would be negative (and a negative distance is not possible).

6 Solve the quadratic equation $2x^2 - x - 5 = 0$, giving your solutions:

(a) in simplest surd form **(b)** correct to 2 decimal places.

a) $\Delta = b^2 - 4ac = 1^2 - 4 \times (-5) \times 2 = 41$ $\Delta > 0$ so 2 solutions

$x = \dfrac{1+\sqrt{41}}{4}$ or $x = \dfrac{1-\sqrt{41}}{4}$

b) $\dfrac{1+\sqrt{41}}{4} \approx 1.85$

$\dfrac{1-\sqrt{41}}{4} \approx -1.35$

7 Find the solutions of the equation $\frac{x}{x+1} - \frac{1}{x+2} = 3$ as surds.

$\Leftrightarrow \dfrac{x(x+2) - (x+1)}{(x+1)(x+2)} = 3 \Leftrightarrow x^2 + 2x - x - 1 = 3(x+1)(x+2)$

$\Leftrightarrow x^2 + x - 1 = 3x^2 + 9x + 6 \Leftrightarrow -2x^2 - 8x - 7 = 0$

$\Leftrightarrow 2x^2 + 8x + 7 = 0 \qquad \Delta = 64 - 4 \times 7 \times 2 = 8 = (2\sqrt{2})^2$

So $\Delta > 0$, $\therefore$ 2 solutions.

$x = \dfrac{-8 + 2\sqrt{2}}{4} = \dfrac{-4 + \sqrt{2}}{2}$

or $x = \dfrac{-8 - 2\sqrt{2}}{4} = \dfrac{-4 - \sqrt{2}}{2}$

8 Expand and simplify $(2x - y)(x^2 - xy + y^2)$.

$(2x - y)(x^2 - xy + y^2) = 2x^3 - 2x^2y + 2y^2x - yx^2 + xy^2 - y^3$

$= 2x^3 - y^3 - 3x^2y + 3xy^2$

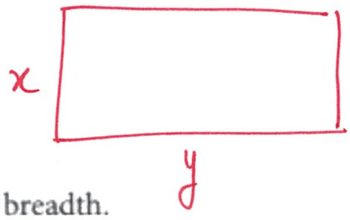
9 Simplify: $\frac{2}{m^2-4}-\frac{1}{m^2-3m+2}$ $= \frac{2}{(m-2)(m+2)} - \frac{1}{(m-2)(m-1)}$

$$= \frac{2(m-1)-(m+2)}{(m-2)(m+2)(m-1)}$$

$$= \frac{2m-2-m-2}{(m-2)(m+2)(m-1)}$$

$$= \frac{m-4}{(m-2)(m+2)(m-1)}$$

11 The perimeter of a rectangle is 18 cm and its area is 20 cm^2.



(a) If the length is x cm, express the breadth in terms of x.

(b) Write the area in terms of x.

(c) Form a quadratic equation in x and solve it to find the length and breadth.

a) $P = 2(x+y) = 18$ $\quad A = 20 = xy.$ $\quad$ so $y = \frac{20}{x}$

$\Rightarrow y = 9 - x$

b) Area $= x(9-x) = 20$

c) $-x^2 + 9x - 20 = 0 \iff x^2 - 9x + 20 = 0$

$\Delta = 9^2 - 4 \times 20 = 1$ $\quad \Delta > 0$ so 2 solutions.

$x = \frac{9+1}{2} = 5$ $\quad$ if so $y = 4$

or $x = \frac{9-1}{2} = 4$ $\quad$ if so $y = 5$

But the length is greater than the width, so length is 5 and width is 4

12 If $n \geq 0$, solve $200 = \frac{n}{2}(6 + 2(n-1))$, rounding your answer to the nearest integer.

$\iff 200 = 3n + n(n-1) \iff 200 = 3n + n^2 - n$

$\iff -n^2 - 2n + 200 = 0 \iff n^2 + 2n - 200 = 0$

$\Delta = 4 - 4 \times (-200) = 804$ $\Delta > 0$ so 2 solutions.

$n = \frac{-2 + \sqrt{804}}{2} = 13.18$ so approx 13

or $n = \frac{-2 - \sqrt{804}}{2}$ but this one is impossible as we are told that $n \geqslant 0$

13 Solve $12x^3 + 12x^2 - 24x = 0$. $\iff x^3 + x^2 - 2x = 0$

$\iff x(x^2 + x - 2) = 0$ so either $x = 0$ or $x^2 + x - 2 = 0$

$\iff x(x-1)(x+2) = 0$

so $x = 0$ or $x = 1$ or $x = -2$

14 Solve $\frac{a-x}{x} = \frac{x}{b-x}$ for x. $\Leftrightarrow (a-x)(b-x) = x^2$

$\Leftrightarrow ab - ax - bx + x^2 = x^2$

$\Leftrightarrow ab - x(a+b) = 0$

$\Leftrightarrow x = \frac{ab}{a+b}$

15 If $x > 0$, solve $22^2 = x^2 + 20^2 - 40x\cos 60°$, giving your answer to the nearest integer.

$\Leftrightarrow 484 = x^2 + 400 - 40x \times \frac{1}{2}$ as $\cos 60 = \frac{1}{2}$

$\Leftrightarrow 84 - x^2 + 20x = 0$

$\Leftrightarrow x^2 - 20x - 84 = 0$

$\Delta = 400 - 4 \times (-84) = 736$ $\Delta > 0$ so 2 solutions.

$x = \frac{20 + \sqrt{736}}{2} = 23.6..$ so 24 to the nearest integer.

or $x = \frac{20 - \sqrt{736}}{2} < 0$ so impossible as we are told that $x > 0$.