Example 14

Two standard dice are rolled and the variable X represents the number of sixes obtained.

Find the expected number of sixes obtained.

Solution

Write the sample space:

$$\left\{\begin{array}{l}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{array}\right\}$$

There are 36 outcomes in the sample space.
Draw a table showing the probability distribution of the random variable.
Leave the probabilities in fraction form with the same denominator to make calculations easier:

x	0	1	2
$P(X=x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Calculate $E(X)$ from first principles. Express the final answer as a fraction in simplest form:

$$\begin{aligned}E(X)&=0\times\frac{25}{36}+1\times\frac{10}{36}+2\times\frac{1}{36}\\&=\frac{12}{36}\\&=\frac{1}{3}\end{aligned}$$

The expected number (or average number) of sixes in two rolls of a standard die is $\frac{1}{3}$.

Example 15

One of the games at the local sporting club's 'Vegas Night' involves rolling a standard six-sided die. If an even number is shown, there is no game charge and the player wins the number of dollars shown on the face of the die. If an odd number is shown, the cost of playing is the number of dollars shown on the face of the die. Let Z stand for the number of dollars received by the player.

(a) Draw up a probability distribution table.

(b) What type of distribution is this?

(c) Find $E(Z)$.

(d) Is the game fair?

Solution

(a) A win could be represented by a positive value and a loss by a negative value.
Draw up the probability distribution table:

z	-1	2	-3	4	-5	6
$P(Z=z)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(b) The probabilities are all equal, so this is a uniform distribution.

(c) The rule for $E(Z)$ cannot be used since the values are not $(1, 2, 3, \ldots, n)$. Express the final answer as a fraction in simplest form:

$$E(Z) = -1 \times \frac{1}{6} + 2 \times \frac{1}{6} - 3 \times \frac{1}{6} + 4 \times \frac{1}{6} - 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

(d) The game is not fair. It is actually in favour of the player since the expected return is \$0.50.