APPROXIMATIONS OF TRIGONOMETRIC FUNCTIONS WHEN x IS SMALL

1 Evaluate the following limits.

(a) $\lim_{x\to 0}\frac{\sin 2x}{x}$ (b) $\lim_{x\to 0}\frac{\sin x}{3x}$ (c) $\lim_{\theta\to 0}\frac{\sin 5\theta}{2\theta}$ (d) $\lim_{x\to 0}\frac{\tan 2x}{2x}$

a) $\lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0} 2\times\left(\frac{\sin 2x}{2x}\right) = 2\times 1 = 2$

as $\lim_{x\to 0}\frac{\sin x}{x} = 1$

b) $\lim_{x\to 0}\frac{\sin x}{3x} = \lim_{x\to 0}\frac{1}{3}\left(\frac{\sin x}{x}\right) = \frac{1}{3}\times\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{3}$

c) $\lim_{\theta\to 0}\frac{\sin 5\theta}{2\theta} = \lim_{\theta\to 0}\frac{\sin 5\theta}{5\theta}\times\frac{5}{2} = \frac{5}{2}\lim_{\theta\to 0}\frac{\sin 5\theta}{5\theta} = \frac{5}{2}$

d) $\lim_{x\to 0}\frac{\tan 2x}{2x} = \lim_{x\to 0}\frac{\sin 2x}{2x}\times\frac{1}{\cos 2x}$

$= \frac{1}{1}\times\lim_{x\to 0}\frac{\sin 2x}{2x}$

$= 1\times 1$

$= 1$

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(e) $\lim_{x\to 0} \frac{\sin\frac{x}{2}}{x}$ (f) $\lim_{x\to 0} \frac{\sin\frac{x}{3}}{3x}$ (g) $\lim_{x\to 0} \frac{\tan 3x}{x}$ (h) $\lim_{x\to 0} \frac{3\sin 2x}{4x}$

e) $\lim_{x\to 0} \frac{\sin(x/2)}{x} = \lim_{x\to 0} \frac{1}{2} \times \frac{\sin(x/2)}{(x/2)} = \frac{1}{2} \lim_{x\to 0} \frac{\sin(x/2)}{x/2} = \frac{1}{2}$

f) $\lim_{x\to 0} \frac{\sin x/3}{3x} = \lim_{x\to 0} \frac{1}{9} \times \frac{\sin(x/3)}{(x/3)} = \frac{1}{9} \times \lim_{x\to 0} \frac{\sin(x/3)}{(x/3)} = \frac{1}{9}$

g) $\lim_{x\to 0} \frac{\tan 3x}{x} = \lim_{x\to 0} 3 \times \frac{\tan 3x}{3x} = 3 \times \lim_{x\to 0} \frac{\tan 3x}{3x} = 3$

h) $\lim_{x\to 0} \frac{3\sin 2x}{4x} = \lim_{x\to 0} \left[\frac{3}{2} \times \frac{\sin 2x}{2x}\right]$

$= \frac{3}{2} \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)$

$= \frac{3}{2} \times 1$

$= \frac{3}{2}$

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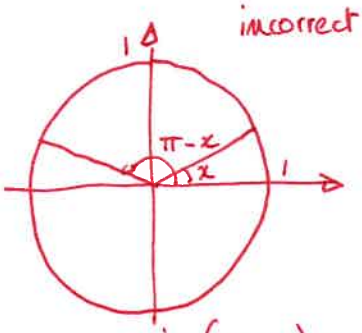
2 Indicate whether each statement is correct or incorrect.

(a) $\sin(\pi - x) = -\sin x$ incorrect (b) $\sin(\pi - x) = \sin x$ correct (c) $\lim_{x\to 0}\frac{\sin(\pi - x)}{x} = -1$ incorrect (d) $\lim_{x\to 0}\frac{\sin(\pi - x)}{x} = 1$ correct



$\sin(\pi - x) = \sin x$

c) $\lim_{x\to 0}\frac{\sin(\pi - x)}{x} = \lim_{x\to 0}\frac{\sin x}{x} = 1$

3 Evaluate the following limits.

(a) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ (b) $\lim_{h\to 0}\frac{\tan 2h}{3h}$ (c) $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$ (d) $\lim_{x\to 0}\frac{\sin^2 x}{x}$

a) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2} = \lim_{x\to 0}\frac{1-\cos 2x}{(2x)^2}\times 4 = 4\times\lim_{x\to 0}\frac{1-\cos 2x}{(2x)^2} = 4\times\frac{1}{2} = 2$

b) $\lim_{R\to 0}\frac{\tan 2R}{3R} = \lim_{R\to 0}\frac{\tan 2R}{2h}\times\frac{2}{3} = \frac{2}{3}\times\lim_{R\to 0}\frac{\tan 2R}{2R} = \frac{2}{3}$

c) $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2} = 1$ as demonstrated in Bosan.

d) $\lim_{x\to 0}\frac{\sin^2 x}{x} = \lim_{x\to 0} x\times\left(\frac{\sin x}{x}\right)^2 = 0\times 1 = 0$

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(e) $\lim_{x\to 0}\frac{\sin(\pi+x)}{x}$ (f) $\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}-x\right)}{x}$ (g) $\lim_{x\to 0}\frac{1-\sin\left(\frac{\pi}{2}-x\right)}{x^2}$ (h) $\lim_{x\to 0}\frac{2\sin\frac{x}{2}}{x}$

e) $\lim_{x\to 0}\frac{\sin(\pi+x)}{x} = \lim_{x\to 0}\frac{-\sin x}{x} = -1 \times \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = -1 \times 1 = -1$

f) $\lim_{x\to 0}\frac{\cos(\pi/2 - x)}{x} = \lim_{x\to 0}\frac{\sin x}{x} = 1$

g) $\lim_{x\to 0}\frac{1-\sin(\pi/2 - x)}{x^2} = \lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$

h) $\lim_{x\to 0}\frac{2\sin(x/2)}{x} = \lim_{x\to 0}\frac{\sin(x/2)}{(x/2)} = 1$