DERIVATIVE OF $e^{f(x)}$

~~DIFFERENTIAL CALCULUS - CHAPTER REVIEW~~

1 Differentiate:

(a) $e^{x^2}+2$ **(b)** $\left(e^x+x^2\right)^4$ **(c)** e^x+ex

(d) $4e^{\cos x}$ **(e)** $e^{\sqrt{x}+1}$ **(f)** $e^{x+\ln x}$

a) $f(x)=e^{x^2}+2$ $u(x)=e^x$ $v(x)=x^2$

$u'(x)=e^x$ $v'(x)=2x$

So $f'(x)=e^{x^2}\times 2x = 2x\,e^{x^2}$

b) $f(x)=[e^x+x^2]^4$ $u(x)=x^4$ $v(x)=e^x+x^2$

$u'(x)=4x^3$ $v'(x)=e^x+2x$

$f'(x)=4[e^x+x^2]^3\times(e^x+2x)$

c) $f(x)=e^x+ex$ $f'(x)=e^x+e$

d) $f(x)=4e^{\cos x}=4g(x)$ $u(x)=e^x$ $v(x)=\cos x$

$u'(x)=e^x$ $v'(x)=-\sin x$

$f'(x)=4\left[e^{\cos x}\times(-\sin x)\right]=-4\sin x\,e^{\cos x}$

e) $f(x)=e^{\sqrt{x}+1}$ $u(x)=e^x$ $v(x)=\sqrt{x}+1=x^{1/2}+1$

$u'(x)=e^x$ $v'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$f'(x)=e^{\sqrt{x}+1}\times\frac{1}{2\sqrt{x}}$

f) $f(x)=e^{x+\ln x}$ $u(x)=e^x$ $v(x)=x+\ln x$

$u'(x)=e^x$ $v'(x)=1+\frac{1}{x}$

So $f'(x)=e^{x+\ln x}\times\left(1+\frac{1}{x}\right)$

$f'(x)=\left(\frac{x+1}{x}\right)e^{x\ln x}$

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2 Differentiate:

(a) $x\,e^{\sin x}$ **(b)** $e^x \log_e x$ **(c)** $e^{\cos(2x+1)}$ **(d)** $1+x+x^2 e^x$

a) $u(x) = x \qquad u'(x) = 1$

$V(x) = e^{\sin x} = g(f(x)) \qquad g(x) = e^x \qquad g'(x) = e^x$

$f(x) = \sin x \qquad f'(x) = \cos x$

So $V'(x) = e^{\sin x} \times (\cos x)$

So $\left(x e^{\sin x}\right)' = 1 \times e^{\sin x} + x\, e^{\sin x} \times (\cos x) = e^{\sin x}\left[x \cos x + 1\right]$

b) $u(x) = e^x \qquad V(x) = \ln x$

$u'(x) = e^x \qquad V'(x) = 1/x$

So $\left(e^x \ln x\right)' = e^x \times \ln x + e^x \times \frac{1}{x} = e^x\left[\ln x + \frac{1}{x}\right]$

c) $u(x) = e^x \qquad V(x) = \cos(2x+1)$

$u'(x) = e^x \qquad V'(x) = -\sin(2x+1) \times 2$

So $\left(e^{\cos(2x+1)}\right)' = e^{\cos(2x+1)} \times \left[-2\sin(2x+1)\right]$

$= -2\sin(2x+1)\, e^{\cos(2x+1)}$

d) Consider $f(x) = x^2 e^x \qquad u(x) = x^2 \qquad u'(x) = 2x$

$V(x) = e^x \qquad V'(x) = e^x$

So $f'(x) = 2x e^x + x^2 e^x = x e^x\left[2 + x\right]$

Therefore $\left[1 + x + x^2 e^x\right]' = 1 + x e^x\left[2 + x\right]$

DERIVATIVE OF $e^{f(x)}$

3 Given $y = \dfrac{100}{1+15e^{-0.5t}}$, find $\dfrac{dy}{dt}$.

$$f(t) = \frac{100}{1+15e^{-0.5t}} = 100\left[1+15e^{-0.5t}\right]^{-1}$$

$$f'(t) = 100 \times (-1)\left[1+15e^{-0.5t}\right]^{-2} \times \left[15 \times (-0.5) \times e^{-0.5t}\right]$$

$$f'(t) = +750\, e^{-0.5t}\left[1+15e^{-0.15t}\right]^{2} = \frac{750\, e^{-0.5t}}{\left[1+15e^{-0.15t}\right]^{2}}$$

5 In statistics, the normal probability density function is given by $f(x) = \dfrac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$. Find $f'(0)$.

$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

consider $g(x) = e^{-x^2/2}$

$u(x) = e^x \qquad u'(x) = e^x$

$v(x) = -\dfrac{x^2}{2} \qquad v'(x) = -\dfrac{2x}{2} = -x$

$$\text{So } \left(e^{-x^2/2}\right)' = e^{-x^2/2} \times (-x) = -x\, e^{-x^2/2}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}} \times \left[-x\, e^{-x^2/2}\right]$$

$$f'(0) = \frac{1}{\sqrt{2\pi}} \times (-0) \times e^{-0^2/2} = 0$$

DERIVATIVE OF $e^{f(x)}$

~~DIFFERENTIAL CALCULUS - CHAPTER REVIEW~~

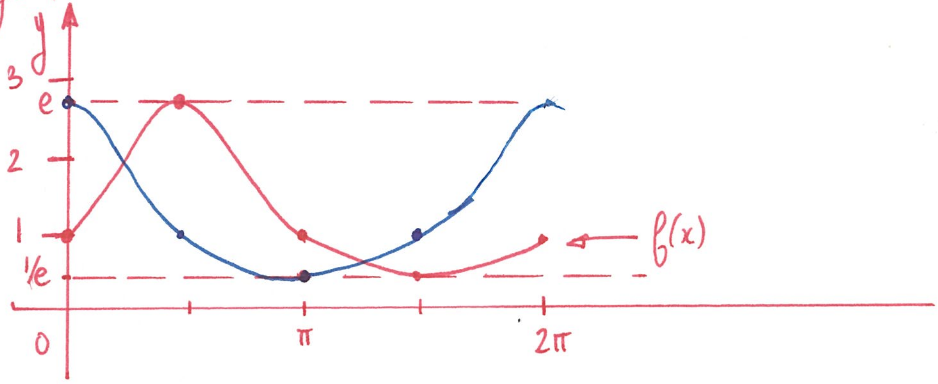
4 (a) Sketch the graphs of $f(x) = e^{\sin x}$ and $g(x) = e^{\cos x}$ on the same diagram for $0 \le x \le 2\pi$, using appropriate technology.

(b) Write the coordinates of their points of intersection (correct to 3 decimal places where necessary). Check your solutions algebraically.

(c) Find the gradient of the tangent to each curve at their points of intersection.

(d) Do the curves intersect at right angles at these points? Justify your answer.

a) $\sin x$ and $\cos x$ take values between -1 and 1, so $f(x)$ and $g(x)$ will take values between e^{-1} and e (i.e. between $\simeq 0.37$ and $\simeq 2.7$)



b) $e^{\sin x} = e^{\cos x}$ when $\sin x = \cos x$ i.e. when $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

i.e $x \simeq 0.785$ or $x \simeq 3.927$

c) $f(x) = e^{\sin x}$ $\quad f'(x) = e^{\sin x} \times \cos x$

$g(x) = e^{\cos x}$ $\quad g'(x) = e^{\cos x} \times (-\sin x)$

at $x = \frac{\pi}{4}$ $\quad f'\left(\frac{\pi}{4}\right) = e^{\sin(\pi/4)} \times \cos\left(\frac{\pi}{4}\right) = e^{\sqrt{2}/2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} e^{\sqrt{2}/2}$

and $g'\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4}\, e^{\cos(\pi/4)} = -\frac{\sqrt{2}}{2} e^{\sqrt{2}/2}$

at $x = \frac{5\pi}{4}$ $\quad f'\left(\frac{5\pi}{4}\right) = e^{\sin(5\pi/4)} \times \cos\left(\frac{5\pi}{4}\right) = e^{-\sqrt{2}/2} \times \left(+\frac{\sqrt{2}}{2}\right)$

and $g'\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{5\pi}{4}\right) e^{\cos(5\pi/4)} = +\frac{\sqrt{2}}{2} e^{-(\sqrt{2}/2)}$

d) $f'\left(\frac{\pi}{4}\right) \times g'\left(\frac{\pi}{4}\right) = \left[\frac{\sqrt{2}}{2} e^{\sqrt{2}/2}\right] \times \left[-\frac{\sqrt{2}}{2} e^{\sqrt{2}/2}\right] \neq -1$ so NO at $x = \pi/4$

$f'\left(\frac{5\pi}{4}\right) \times g'\left(\frac{5\pi}{4}\right) = \left[\frac{\sqrt{2}}{2} e^{-\sqrt{2}/2}\right]\left[\frac{\sqrt{2}}{2} e^{-\sqrt{2}/2}\right] \neq -1$ so NO at $x = \frac{5\pi}{4}$