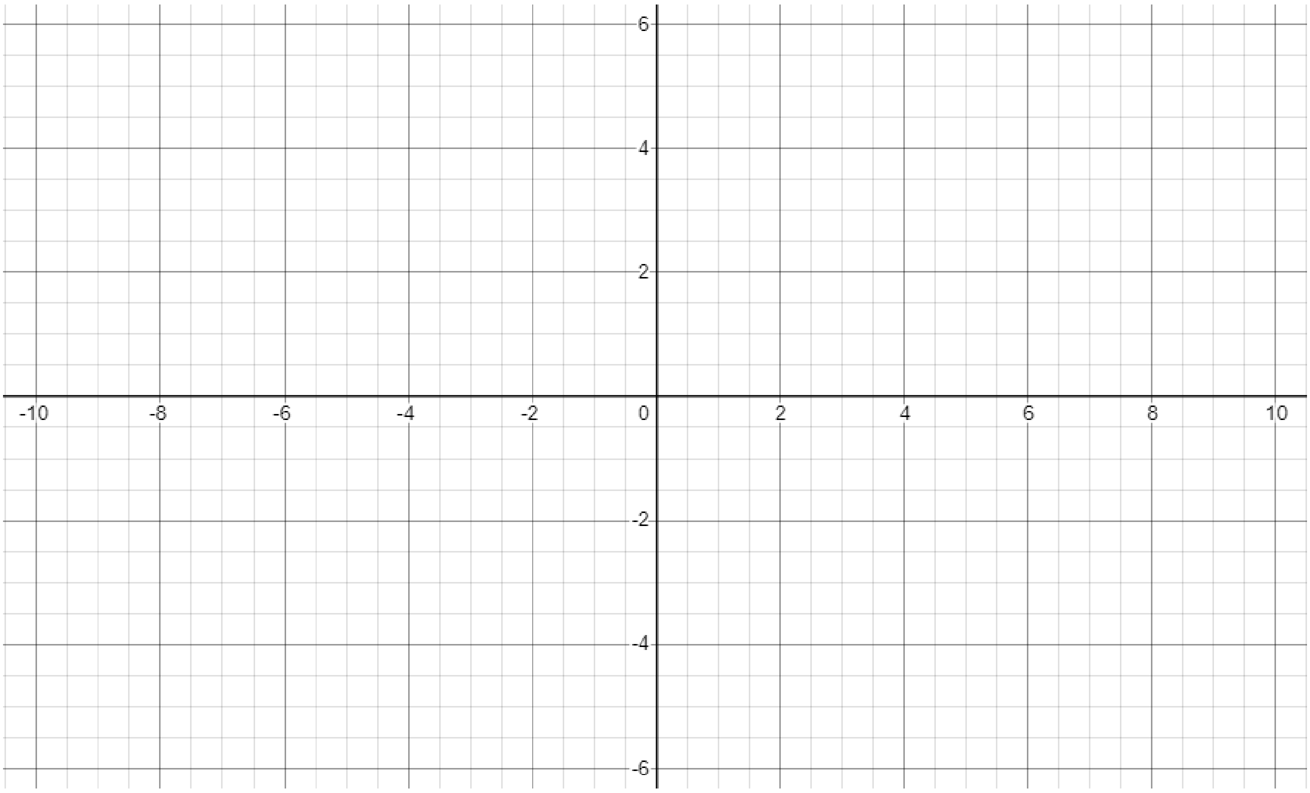
GEOMETRICAL REPRESENTATION OF A COMPLEX NUMBER

1 If $z = 2 + i$ and $w = -3 - 4i$, represent each of the following on the complex plane.

(a) z **(b)** $\overline{z}$ **(c)** $z\overline{z}$ **(d)** $3z$ **(e)** $-2z$ **(f)** $\frac{1}{z}$ **(g)** $z + w$
(h) $-w$ **(i)** $z - w$ **(j)** z^2 **(k)** $\text{Re}(z)$ **(l)** $\text{Im}(z)$



2 If $z = 2\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)$, then $z^4 = \ldots$

A $16\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)$ **B** $16\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

C $16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ **D** $16\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

3 If $z = \overline{z}$, then $\arg z = \ldots$

A π **B** $\frac{\pi}{2}$ **C** 0 **D** 0 or π

4 Express each of the following in mod–arg form. (Give the argument in radians and in exact form.)

(a) $2 - 2i$ **(b)** $-\sqrt{3} + i$ **(c)** $-6 - 6i$ **(d)** $4i$ **(e)** -4

(f) $-3-\sqrt{3}i$ **(g)** $2\sqrt{3}-2i$ **(h)** $\sqrt{2}+\sqrt{2}i$

6 For each of the following, find both zw and $\frac{z}{w}$ in mod–arg form.

(a) $z = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, $w = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(b) $z = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, $w = 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

7 If $z = x + iy$, prove the following.

(a) $|z| = |\overline{z}|$ **(b)** $z\overline{z} = |z|^2$ **(c)** $z + \frac{|z|^2}{z} = 2\text{Re}(z)$

11 If $z = r(\cos\theta + i\sin\theta)$, show that $\frac{z}{z^2 + r^2}$ is real.

14 Use de Moivre's theorem to prove that the conjugate of a power is equal to the power of the conjugate, i.e. let $z = r(\cos\theta + i\sin\theta)$ and prove that $\overline{z^n} = (\overline{z})^n$.

15 We have already proved (earlier and in question **14**) that:

- $z + \overline{z} = 2\text{Re}(z)$ and $z - \overline{z} = 2\text{Im}(z) \times i$
- the conjugate of a sum is equal to the sum of the conjugates
- the conjugate of a difference is equal to the difference of the conjugates
- the conjugate of a product is equal to the product of the conjugates
- the conjugate of a quotient is equal to the quotient of the conjugates
- the conjugate of a power is equal to the power of the conjugate.
- It is also obvious that the conjugate of a real number is itself, i.e. if $z = x + 0i$ then $\overline{z} = x - 0i = z$.

Use these properties of conjugates to answer the following.

(a) Show that $z^n + (\overline{z})^n = 2\text{Re}(z^n)$.

(b) Simplify $(1 + \sqrt{3}i)^{10} + (1 - \sqrt{3}i)^{10}$.

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16 Consider the cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ for which all the coefficients a, b, c and d are real. Let the complex number z be a root of the equation $P(x) = 0$. Show that $\overline{z}$ is also a root of $P(x) = 0$.