BASICS — PARABOLAS of the form $y = ax^2 + bx + c$

Equations of the form

$$y = ax^2 + bx + c$$

represent parabolas.

Domain: all real x

Range: restricted
(look for y-coordinate of vertex).

Graphing Parabolas

When graphing parabolas:

- Label the axes, origin, and equation of each curve.
- you must show the x- and y- intercepts and the VERTEX

Features of $y = ax^2 + bx + c$

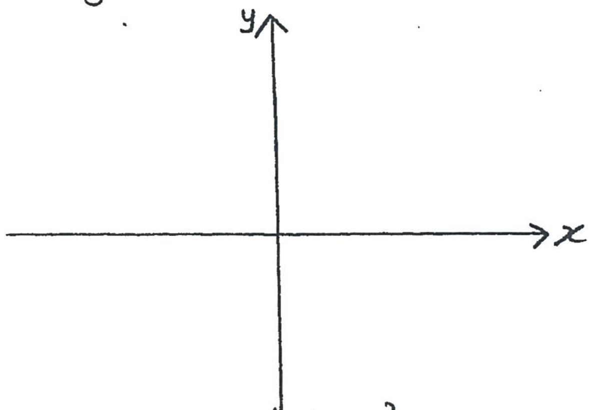
- If $a > 0$, the parabola is concave up
- If $a < 0$, the parabola is concave down
- The larger the value of $|a|$, the narrower the parabola.
- Equation of the axis of symmetry is:

$$x = \frac{-b}{2a}$$

Examples: The BASICS

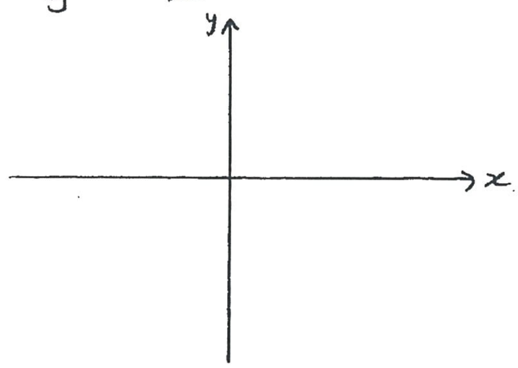
① Concavity

a) $y = x^2$



Function or relation?
Domain :
Range :

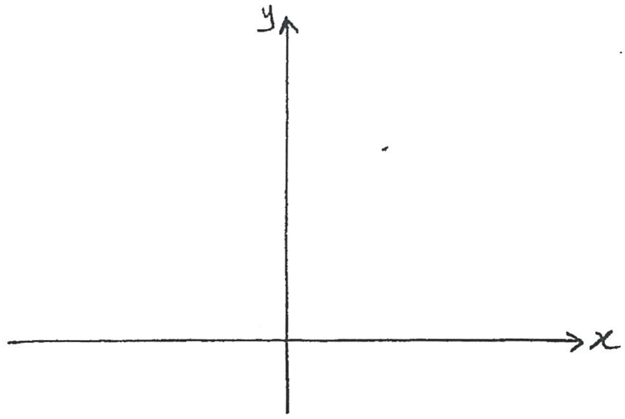
b) $y = -x^2$



Function or relation?
Domain :
Range :

② Curvature

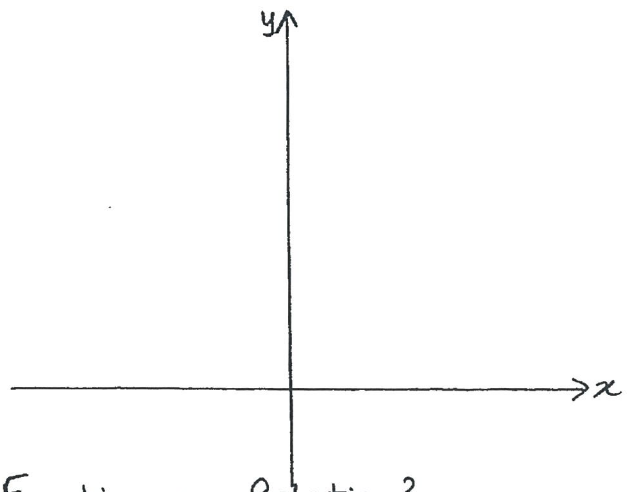
$y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$



③ Shifting Up/Down y-axis

For $y = ax^2 + c$, the value of c determines the y-intercept.

a) $y = x^2 + 2$

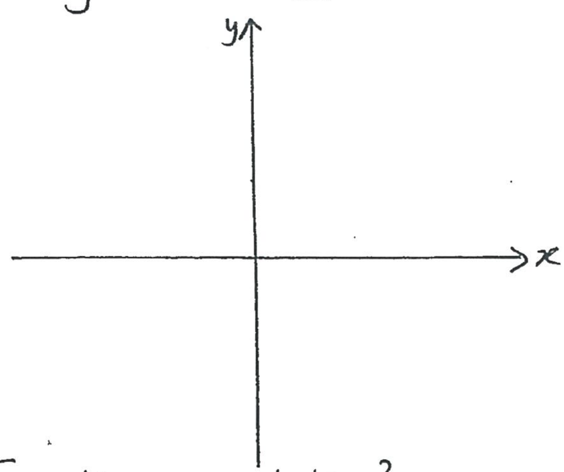


Function or Relation?

Domain:

Range:

b) $y = x^2 - 2$

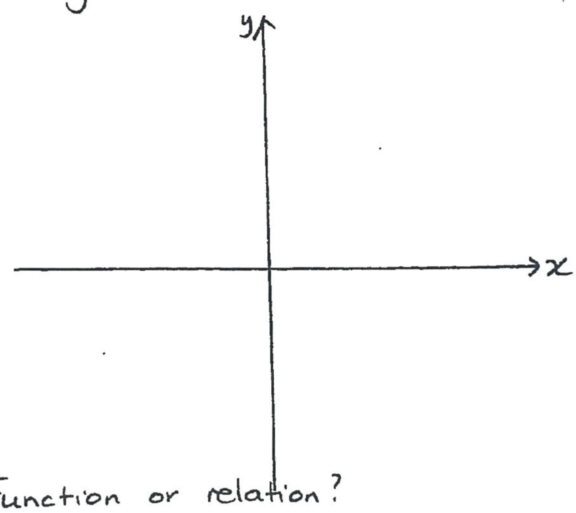


Function or relation?

Domain:

Range:

c) $y = 2 - x^2$

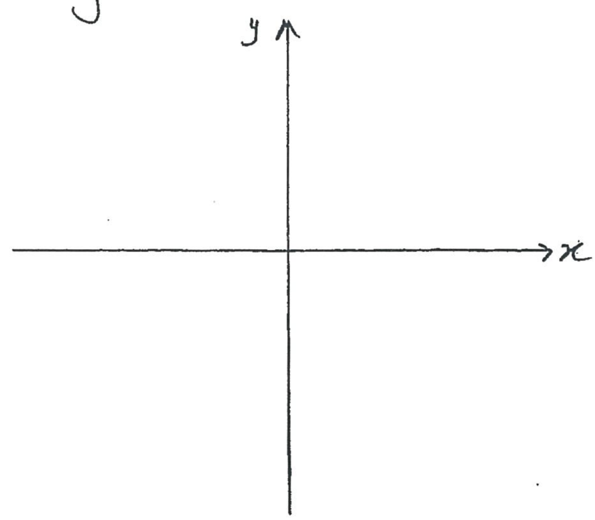


Function or relation?

Domain:

Range:

d) $y = -x^2 - 2$



Function or relation?

Domain:

Range:

BASICS PARABOLAS OF THE FORM $y = ax^2 + bx + c$

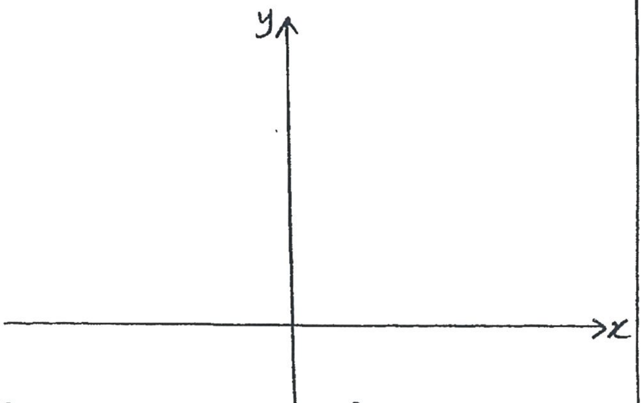
4. Shifting along the x-axis

If k is a constant,

$y = (x + k)^2 \Rightarrow$ shift to LEFT

$y = (x - k)^2 \Rightarrow$ shift to RIGHT

a) $y = (x + 2)^2$



Function or relation?

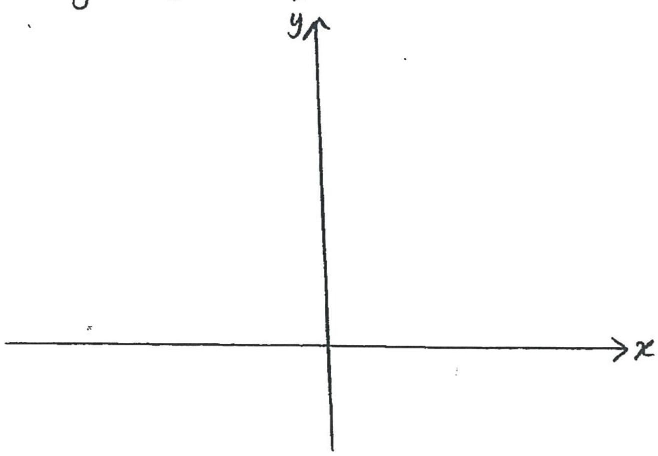
Domain:

Range:

Axis of Symmetry:

Vertex:

b) $y = (x - 2)^2$



Function or relation?

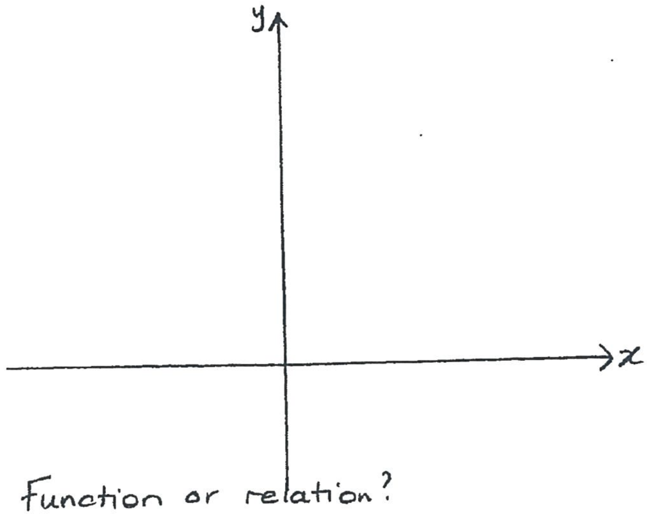
Domain:

Range:

Axis of Symmetry:

Vertex:

c) $y = (x - 2)^2 + 1$



Function or relation?

Domain:

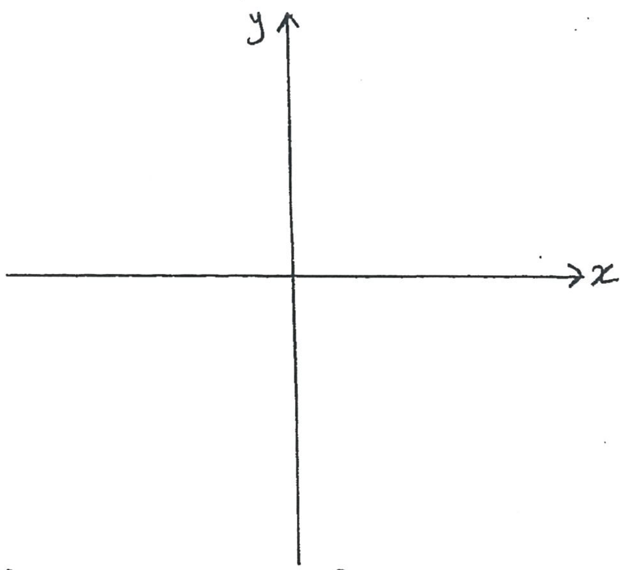
Range:

Axis of Symmetry:

Vertex:

d) $y = (2 - x)^2$

concave up or down?



Function or relation?

Domain:

Range:

Axis of Symmetry:

Vertex:

PARABOLAS OF THE FORM $y=ax^2+bx+c$

Graphing more complex parabolas

To graph more complex parabolas, follow these steps:

1. Determine if the curve is concave up or down.
2. Factorise the equation of the curve. (if possible)
3. Find any x-intercepts, by solving $y=0$.
4. Find the axis of symmetry.
   *By inspection : halfway between the x-values obtained in (3).
   *By formula : $x=\frac{-b}{2a}$
5. Find the coordinates of the vertex. (Vertex lies on axis of symmetry).
6. Find y-intercept, by letting $x=0$.

Note: [A] If the equation of the curve can not be easily factorised to find the solutions when $y=0$, use the quadratic formula instead

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

[B] If there are no solutions when $y=0$, then there are no x-intercepts, and you must use the axis of symmetry, vertex and y-intercept to draw the graph. (see page 11)

Examples: Sketch :

① $y=x^2-4x$

*concave up/down? ______

* factorise : ______

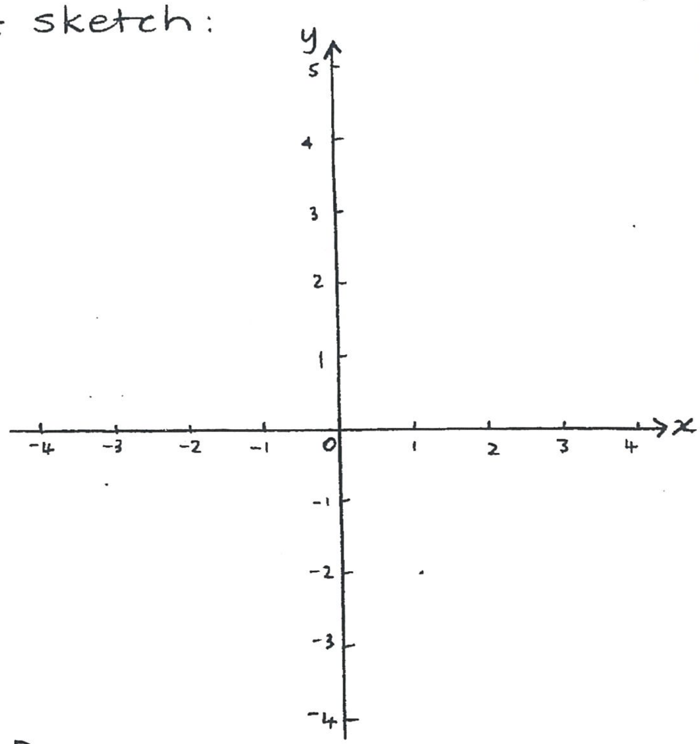
* x-intercepts (solve for $y=0$)

* axis of symmetry:

* vertex :

* y-intercept:

* sketch :



Domain:

Range:

PARABOLAS OF THE FORM $y=ax^2+bx+c$

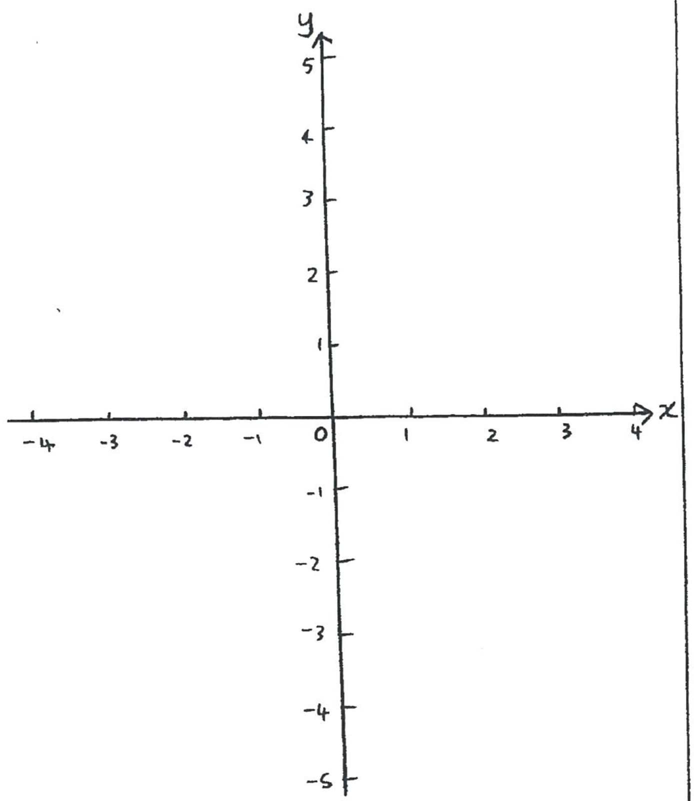
② $y = x^2 - 2x - 3$

* concave ______
* factorise ________________
* x-intercepts (solve for $y=0$)

* axis of symmetry

* vertex

* y-intercept



Domain:

Range:

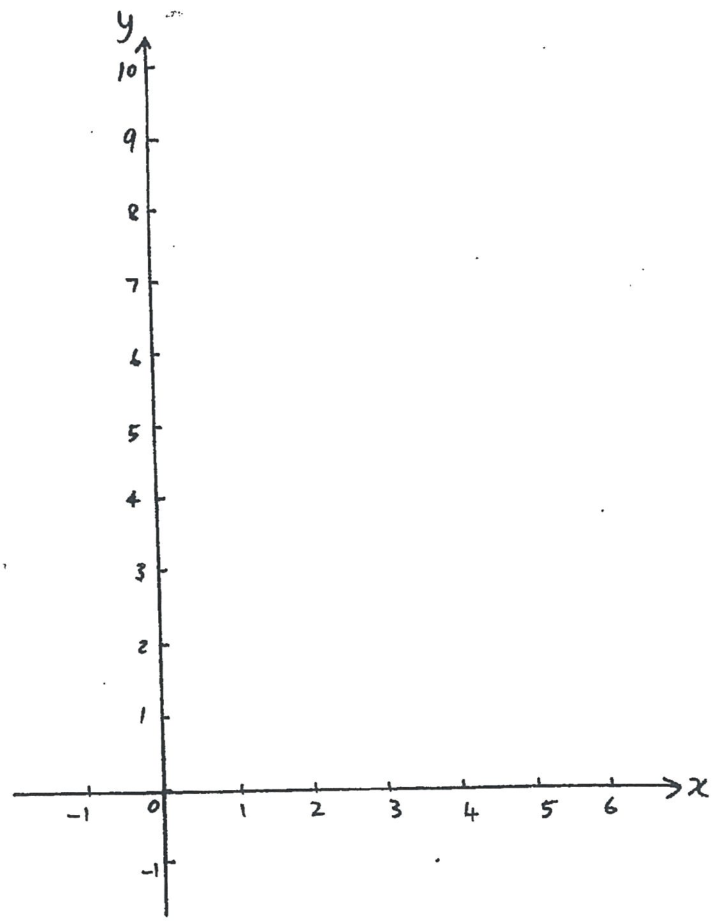
③ $y = x^2 - 6x + 9$

* concave: ________________
* factorise: ________________
* x-intercepts (solve for $y=0$)

* axis of symmetry:

* vertex

* y-intercept



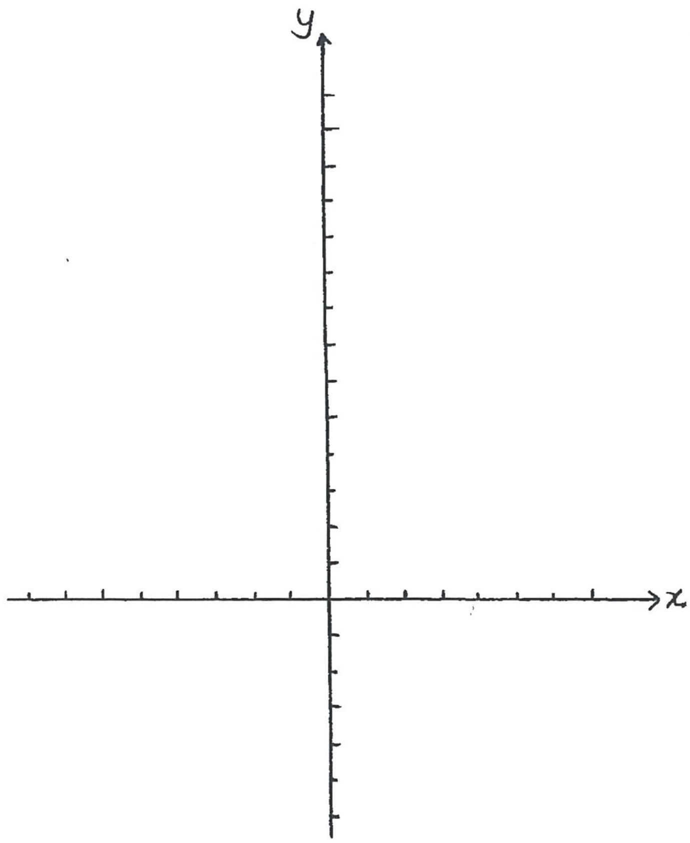
Domain:

Range:

PARABOLAS OF THE FORM $y = ax^2 + bx + c$

4 $y = 6 - 5x - x^2$

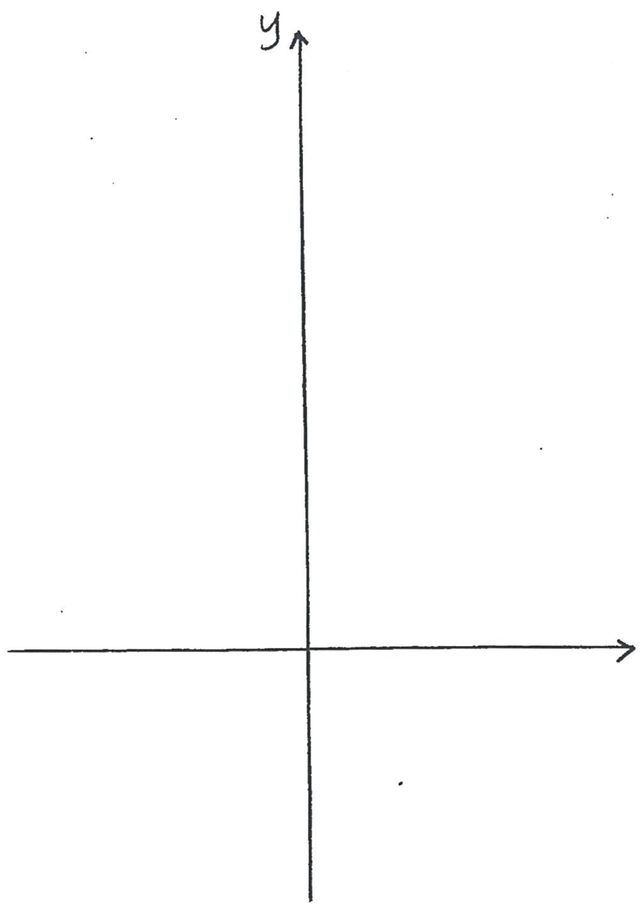
* concave ________
* factorise ______________
* x-intercepts (solve for $y = 0$)
* axis of symmetry
* vertex
* y-intercept



Domain:

Range:

5 $y = 9 - x^2$



Domain:

Range:

6. $y = x^2 - 4x + 5$

Minimum & Maximum Values of Parabolas

▶ Minimum values

If the coefficient of x^2 is positive, the graph is concave upwards, and it has a minimum value.

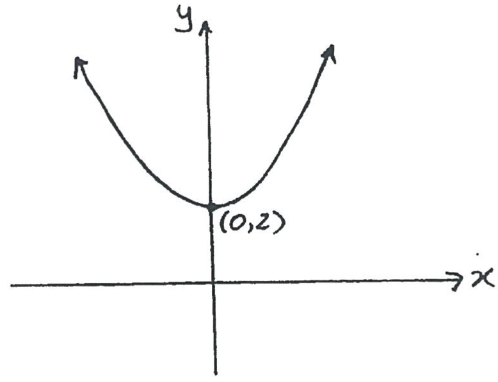
This minimum value is the y-coordinate of the vertex.

▶ Maximum values

If the coefficient of x^2 is negative, the graph is concave downwards, and it has a maximum value.

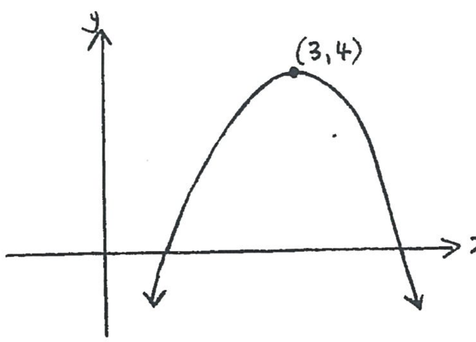
This maximum value is the y-coordinate of the vertex.

1.



Minimum value is ______

2.



maximum value is ______

QUADRATIC FUNCTIONS (Parabolas)

Quadratic functions are of the form: $f(x) = ax^2 + bx + c$ where a, b, c are constants; this is called the "**general form**" of a quadratic function. These curves are called "**parabolas**". If $a > 0$, the parabola is **concave up**; if $a < 0$, the parabola is **concave down.**

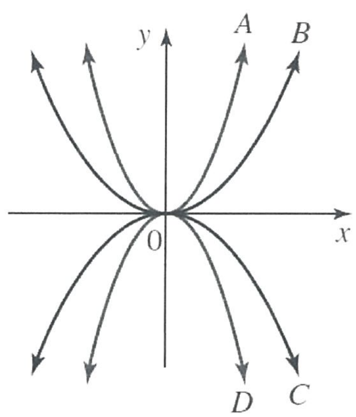
In fact, the general form of a parabola (i.e. $f(x) = ax^2 + bx + c$, as noted above) can be rearranged as two possible other forms:

1) $f(x) = a(x-h)^2 + k$ this form is called the "**vertex form**" as (h, k) are the coordinates of the vertex of the parabola.

2) $f(x) = a(x-r_1)(x-r_2)$ this form is called the "**intercept form**" as $(r_1, 0)$ and $(r_2, 0)$ are the coordinates of the x-intercepts of the parabola.

When asked to find the equation of a parabola from a graph, we use one of these 3 forms, depending on the information provided by the graph.

10



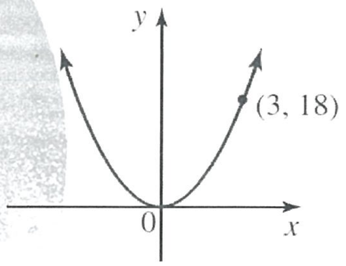
Four parabolas $y = x^2$, $y = -x^2$, $y = 2x^2$ and $y = -2x^2$ have been drawn on the same number plane. State the graph whose equation is:

a $y = x^2$ **b** $y = -x^2$

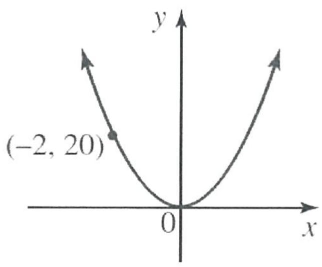
c $y = 2x^2$ **d** $y = -2x^2$

11 The curves below are parabolas with equations of the form $y = ax^2$, where a is a constant. For each curve, find the value of a and hence determine its equation.

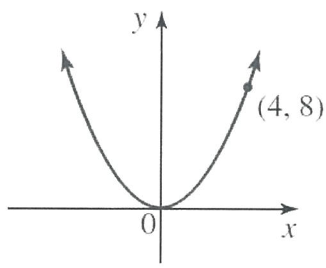
a



b



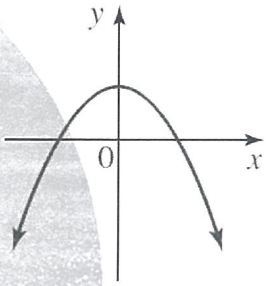
c



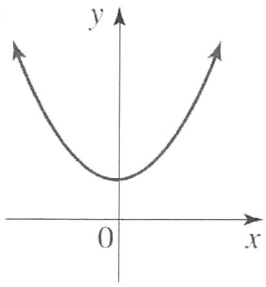
6 Match each of these equations with one of the graphs below.

- $y = x^2 + 3$
- $y = x^2 - 3$
- $y = 3 - x^2$
- $y = -x^2 - 3$

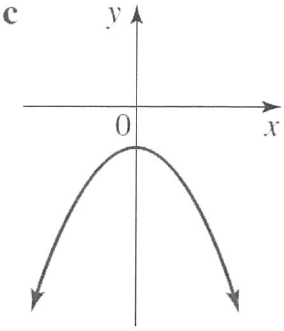
a



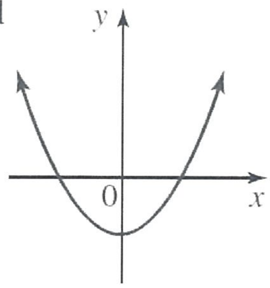
b



c



d

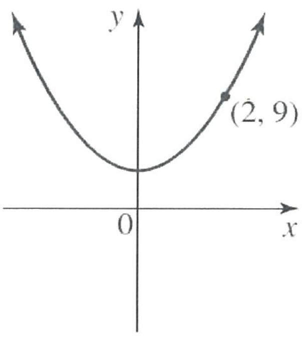


7 Find the equation of the new parabola if the curve $y = x^2 + 2$ is translated:

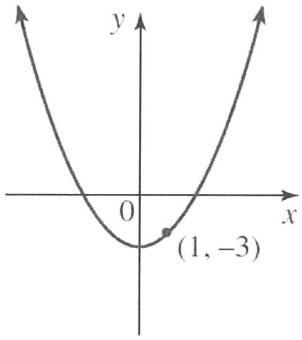
a 6 units up **b** 2 units down **c** 5 units down

10 The curves below are parabolas with equations of the form $y = x^2 + c$ or $y = -x^2 + c$. For each curve, find the value of c and hence determine its equation.

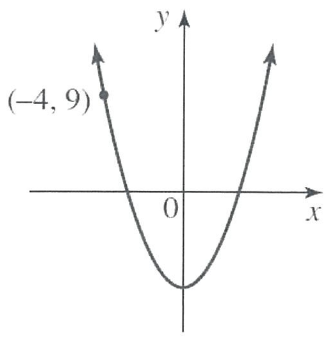
a



b

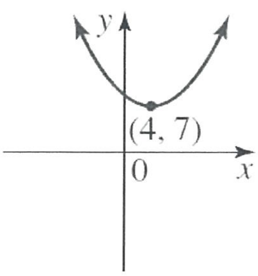


c

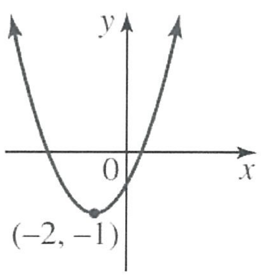


9 Find the equation of each curve in the form $y = a(x - h)^2 + k$, where $a = 1$ or -1.

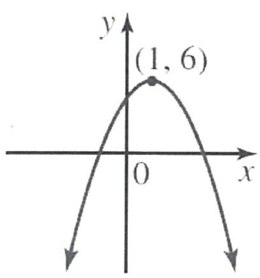
a



b

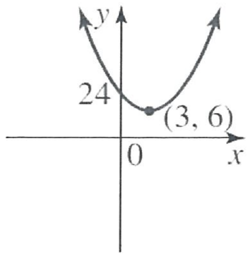


c

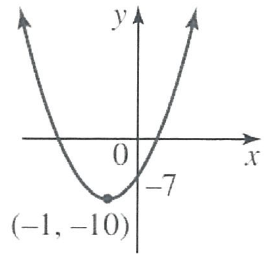


10 Find the equation of each parabola in the form $y = a(x - h)^2 + k$.

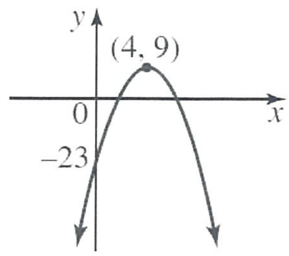
a



b

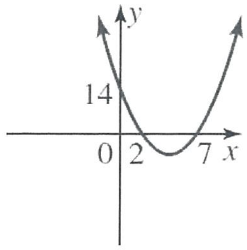


c

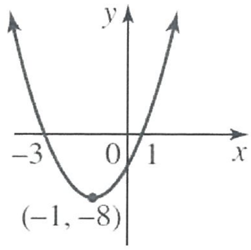


10 Find the equation of each parabola in the form $y = k(x - a)(x - b)$.

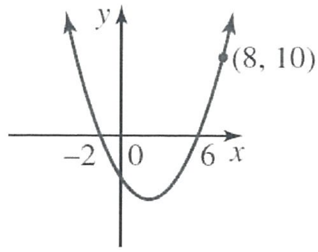
a



b

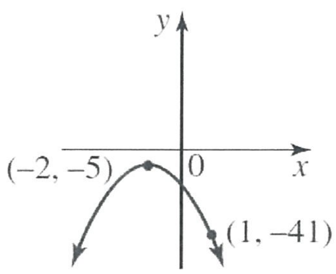


c

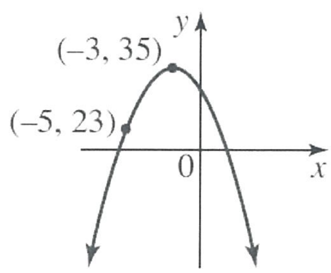


Find the equation of each parabola, using either the standard, vertex or intercept forms.

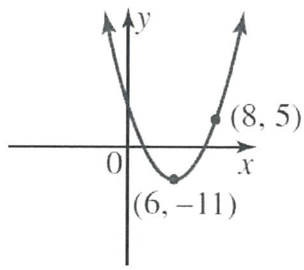
d



e



f



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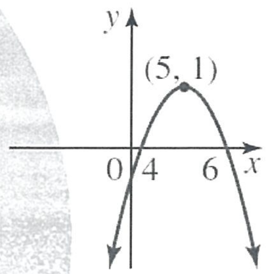

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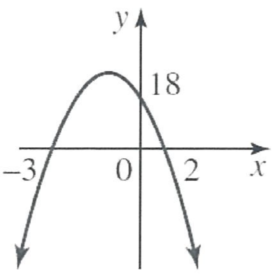

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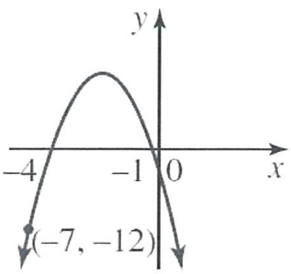
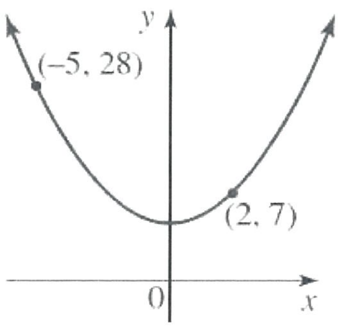
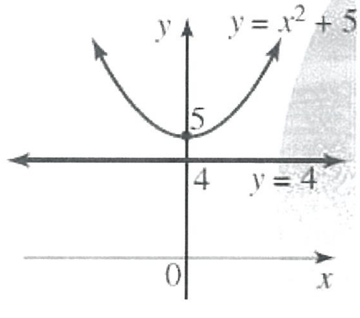


12



The parabola shown has an equation of the form $y = ax^2 + c$. Form a pair of simultaneous equations and hence find the equation of the parabola.

13 What would be the equation of the new parabola if the curve $y = x^2 + 5$ is reflected in the line $y = 4$?



11 Find the equation of the parabola that passes through the points (0, 4), (1, 5) and (−3, 25). [Hint: Start $y = ax^2 + bx + c$ and find c first.]

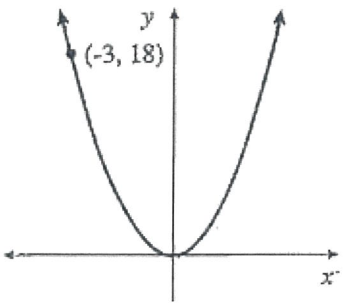
Transform the equation of the parabola from standard form to vertex form by completing the square.

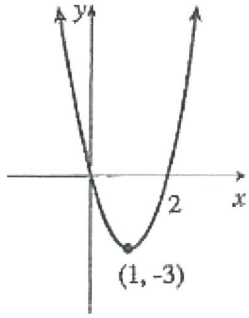
a) $y = 2x^2 - 4x + 5$

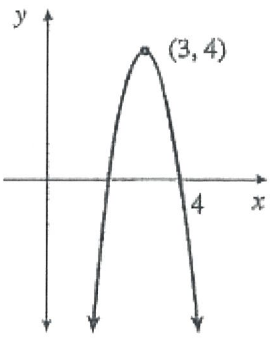
b) $y = -3x^2 - 12x - 7$

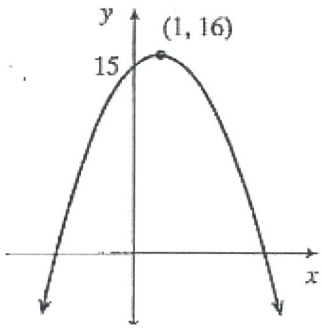
Find the equation of each parabola

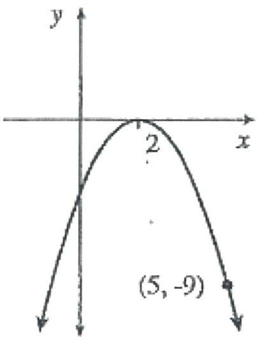
1

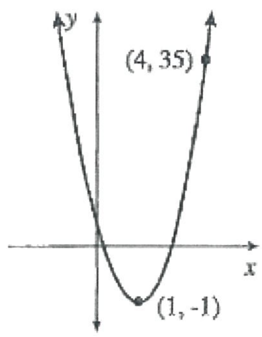


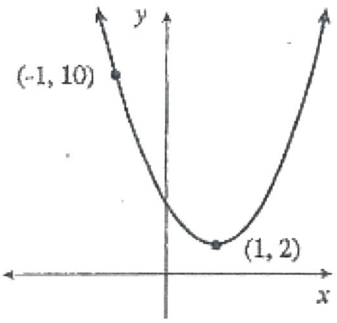


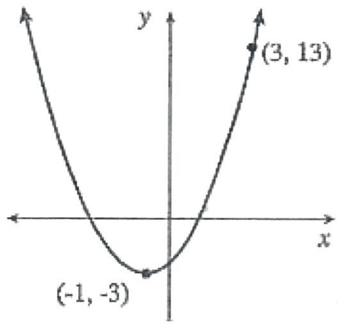


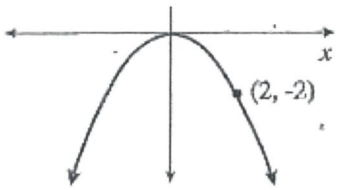


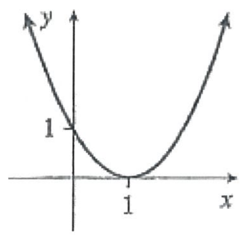












11 A member of an indoor cricket team, playing a match in a gymnasium, hits a ball that follows a path given by $y = -0.1x^2 + 2x + 1$, where y is the height above ground, in metres, and x is the horizontal distance travelled by the ball.

The ceiling of the gymnasium is 10.6 metres high. Will this ball hit the roof? Explain.