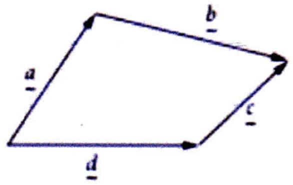
VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

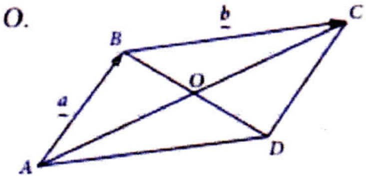
1 Four vectors, $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$ and $\underset{\sim}{d}$, are shown in the diagram. Which one of the following statements is true?



A $\underset{\sim}{a}+\underset{\sim}{c}=\underset{\sim}{b}+\underset{\sim}{d}$ $\boxed{\textbf{B}}$ $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}+\underset{\sim}{d}$

C $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}+\underset{\sim}{d}=\underset{\sim}{0}$ **D** $\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{a}+\underset{\sim}{d}$

2 In the parallelogram $ABCD$ shown, the point of intersection of the diagonals is O. The vector $\overrightarrow{OD}$ is equal to:



A $\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b})$ **B** $\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$ $\overrightarrow{OD}=\frac{1}{2}\overrightarrow{BD}$

C $\frac{1}{2}\underset{\sim}{b}-\underset{\sim}{a}$ $\boxed{\textbf{D}}$ $\frac{1}{2}(\underset{\sim}{b}-\underset{\sim}{a})$ $\overrightarrow{OD}=\frac{1}{2}(\overrightarrow{BA}+\overrightarrow{AD})$

So $\overrightarrow{OD}=\frac{1}{2}(-\underset{\sim}{a}+\underset{\sim}{b})=\frac{1}{2}(\underset{\sim}{b}-\underset{\sim}{a})$

3 If vector $\underset{\sim}{a}$ is represented by the ordered pair $(-2, 3)$, then the vector $-3\underset{\sim}{a}$ is represented by the ordered pair:

A $(-6, 9)$ **B** $(-6, -9)$ $\boxed{\textbf{C}}$ $(6, -9)$ **D** $(6, 9)$

4 The vector that runs from the point $(-3, 1)$ to the point $(3, -2)$ can be represented by the column vector:

A $\begin{pmatrix}6\\3\end{pmatrix}$ $\boxed{\textbf{B}}$ $\begin{pmatrix}6\\-3\end{pmatrix}$ **C** $\begin{pmatrix}-6\\3\end{pmatrix}$ **D** $\begin{pmatrix}0\\-3\end{pmatrix}$

5 Which one of the following vectors is parallel to the vector $\underset{\sim}{f}=-6\underset{\sim}{i}+4\underset{\sim}{j}$?

$\boxed{\textbf{A}}$ $\underset{\sim}{a}=24\underset{\sim}{i}-16\underset{\sim}{j}$ **B** $\underset{\sim}{b}=3\underset{\sim}{i}+2\underset{\sim}{j}$ **C** $\underset{\sim}{c}=-24\underset{\sim}{i}-16\underset{\sim}{j}$ **D** $\underset{\sim}{d}=-3\underset{\sim}{i}-2\underset{\sim}{j}$

as $24=(-4)\times(-6)$ and $-16=(-4)\times 4$

6 Which one of the following vectors is parallel to the vector $\underset{\sim}{a}=-3\underset{\sim}{i}+7\underset{\sim}{j}$ and has a magnitude of $2\sqrt{58}$?

A $-24\underset{\sim}{i}+28\underset{\sim}{j}$ **B** $-\frac{3}{2}\underset{\sim}{i}+\frac{7}{2}\underset{\sim}{j}$ **C** $3\underset{\sim}{i}-7\underset{\sim}{j}$ $\boxed{\textbf{D}}$ $-6\underset{\sim}{i}+14\underset{\sim}{j}$

$|\vec{A}|=\sqrt{24^2+28^2}=\sqrt{1360}=\sqrt{2^4\times5\times17}=4\sqrt{85}$ so not $\boxed{A}$

$|\vec{B}|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{7}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{49}{4}}=\sqrt{\frac{58}{4}}=\frac{1}{2}\sqrt{58}$ so not $\boxed{B}$

$|\vec{C}|=\sqrt{3^2+7^2}=\sqrt{9+49}=\sqrt{58}$ so not $\boxed{C}$

$|\vec{D}|=\sqrt{6^2+14^2}=\sqrt{36+196}=\sqrt{232}=\sqrt{2^3\times29}=2\sqrt{58}$ so $\boxed{D}$

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7 Given position vectors $\overrightarrow{OA} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{OB} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$, what is the value of $\left|\overrightarrow{AB}\right|$?

A $\sqrt{2}$ $\qquad$ $\boxed{\textbf{B}}$ $5\sqrt{2}$ $\qquad$ **C** $2\sqrt{5}$ $\qquad$ **D** $7\sqrt{2}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -(-3\vec{i} + 4\vec{j}) + 4\vec{i} + 3\vec{j} = 7\vec{i} - \vec{j}$

$|\overrightarrow{AB}| = \sqrt{7^2 + 1^2} = \sqrt{49+1} = \sqrt{50} = \sqrt{2 \times 5^2} = 5\sqrt{2}$

8 If $\underset{\sim}{a} = -4\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$, then $2\underset{\sim}{a} - \underset{\sim}{b}$ is:

A $-5\underset{\sim}{i} - 2\underset{\sim}{j}$ $\qquad$ **B** $-6\underset{\sim}{i} + 10\underset{\sim}{j}$ $\qquad$ $\boxed{\textbf{C}}$ $-9\underset{\sim}{i} + 8\underset{\sim}{j}$ $\qquad$ **D** $-10\underset{\sim}{i} - 4\underset{\sim}{j}$

$2\vec{a} - \vec{b} = 2(-4\vec{i} + 2\vec{j}) - (\vec{i} - 4\vec{j})$

$2\vec{a} - \vec{b} = -8\vec{i} + 4\vec{j} - \vec{i} + 4\vec{j} = -9\vec{i} + 8\vec{j}$

9 What is the magnitude of the vector $\underset{\sim}{a} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$?

A 2 $\qquad$ **B** $2\sqrt{3}$ $\qquad$ $\boxed{\textbf{C}}$ $2\sqrt{5}$ $\qquad$ **D** 20

$|\vec{a}| = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = \sqrt{2^2 \times 5} = 2\sqrt{5}$

10 Which of the following vectors is parallel to the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$ and has a magnitude of $2\sqrt{13}$?

A $-4\underset{\sim}{i} + 6\underset{\sim}{j}$ $\qquad$ **B** $4\underset{\sim}{i} - 6\underset{\sim}{j}$ $\qquad$ **C** $6\underset{\sim}{i} + 9\underset{\sim}{j}$ $\qquad$ $\boxed{\textbf{D}}$ $-4\underset{\sim}{i} - 6\underset{\sim}{j}$

$|\vec{A}| = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$|\vec{B}| = \sqrt{16+36} = 2\sqrt{13}$

$|\vec{C}| = \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}$ so not $\boxed{C}$

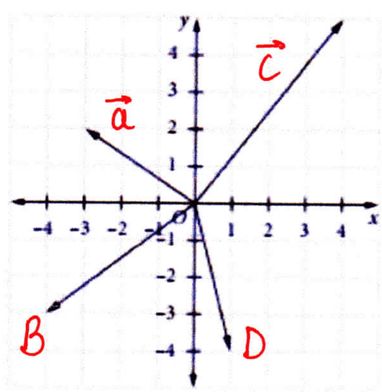
$|\vec{D}| = 2\sqrt{13}$

Vector $\vec{D}$ as $-4 = (-2) \times 2$

and $-6 = (-2) \times 3$

19 Label the following vectors that have been drawn on the Cartesian plane:

- $\underset{\sim}{a}$ the position vector of $(-3, 2)$
- $\overrightarrow{OB}$ where B is $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$
- $\underset{\sim}{c}$ the position vector of $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$
- $\overrightarrow{OD}$ where D is $(1, -4)$



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26 Find the exact values of the unknown pronumerals in the following vector equations.

(a) $(2a-3b)\underset{\sim}{i}-2b\underset{\sim}{j}=5\underset{\sim}{i}-12\underset{\sim}{j}$ **(b)** $(2f+5)\underset{\sim}{i}+(8-7g)\underset{\sim}{j}=f(3\underset{\sim}{i}-2\underset{\sim}{j})+2g(\underset{\sim}{i}+4\underset{\sim}{j})$

(c) $(a^2-9a)\underset{\sim}{i}+(2b^3+1)\underset{\sim}{j}=10\underset{\sim}{i}-5\underset{\sim}{j}$ (list multiple solutions)

a) $\begin{cases}2a-3b=5\\-2b=-12\end{cases} \Leftrightarrow \begin{cases}b=6\\2a-3\times 6=5\end{cases} \Leftrightarrow \begin{cases}b=6\\2a=23\end{cases} \Leftrightarrow \begin{cases}a=23/2\\b=6\end{cases}$

b) $\begin{cases}2f+5=3f+2g\\8-7g=-2f+8g\end{cases} \Leftrightarrow \begin{cases}5=f+2g\\8=-2f+15g\end{cases} \Leftrightarrow \begin{cases}10=2f+4g\\8=-2f+15g\end{cases}$

$\Leftrightarrow \begin{cases}5=f+2g\\18=19g\end{cases} \Leftrightarrow \begin{cases}g=18/19\\f=5-2\times\frac{18}{19}\end{cases} \Leftrightarrow \begin{cases}g=18/19\\f=59/19\end{cases}$

c) $\begin{cases}a^2-9a=10\\2b^3+1=-5\end{cases} \Leftrightarrow \begin{cases}a^2-9a-10=0\\b^3=-3\end{cases} \Leftrightarrow \begin{cases}b=-\sqrt[3]{3}\\a^2-9a-10=0\end{cases}$

roots are $a_1=-1$ and $a_2=10$

so $a_1=-1$ or $a_2=10$, and $b=-\sqrt[3]{3}$

27 Consider the vector $\underset{\sim}{a}=-9\underset{\sim}{i}-3\underset{\sim}{j}$.

(a) Find $\hat{\underset{\sim}{a}}$. **(b)** Find the vector $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ with a magnitude of 5.

a) $|\vec{a}|=\sqrt{9^2+3^2}=\sqrt{81+9}=\sqrt{90}=3\sqrt{10}$

So $\hat{\underset{\sim}{a}}=\frac{1}{3\sqrt{10}}\left[-9\vec{i}-3\vec{j}\right]=-\frac{1}{\sqrt{10}}\left[3\vec{i}+\vec{j}\right]$

b) $\vec{b}=5\times\left[-\frac{1}{\sqrt{10}}(3\vec{i}+\vec{j})\right]=-\sqrt{\frac{5}{2}}(3\vec{i}+\vec{j})$

28 Find the scalar product $\underset{\sim}{a} \bullet \underset{\sim}{b}$, given the following pairs of vectors.

(a) $\underset{\sim}{a} = -4\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 7\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = 3\underset{\sim}{i} - 7\underset{\sim}{j}$ and $\underset{\sim}{b} = 6\underset{\sim}{i} - \underset{\sim}{j}$

a) $\vec{a}.\vec{b} = (-4\vec{i}+\vec{j}).(2\vec{i}+7\vec{j}) = (-4)\times 2 + 1\times 7 = -8+7 = -1$

b) $\vec{a}.\vec{b} = (3\vec{i}-7\vec{j}).(6\vec{i}-\vec{j}) = 3\times 6 + (-7)\times(-1)$

$\vec{a}.\vec{b} = 18+7 = 25$

29 Calculate the scalar product and hence show that the vectors $\underset{\sim}{a} = -3\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{b} = 10\underset{\sim}{i} + 6\underset{\sim}{j}$ are perpendicular.

$\vec{a}.\vec{b} = (-3\vec{i}+5\vec{j}).(10\vec{i}+6\vec{j})$

$\vec{a}.\vec{b} = (-3)\times(10) + 5\times 6 = -30+30 = 0$

$\vec{a}.\vec{b} = 0$ hence $\vec{a}$ and $\vec{b}$ are perpendicular

30 For each of the following pairs of vectors, find the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(a) $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 6\underset{\sim}{i} + 3\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = -5\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 7\underset{\sim}{j}$

a) $\vec{a}.\vec{b} = (3\vec{i}-4\vec{j}).(6\vec{i}+3\vec{j}) = 18-12 = 6$

~~and~~ $\cancel{|\vec{a}||\vec{b}| = \sqrt{3^2+4^2}\times\sqrt{6^2+3^2} = \sqrt{25}\sqrt{45} = \sqrt{1125} = \sqrt{3^2\times 5^3} = 15\sqrt{5}}$

The scalar projection of $\vec{a}$ onto $\vec{b}$ is $\vec{a}.\dfrac{\vec{b}}{|\vec{b}|}$.

$|\vec{b}| = \sqrt{6^2+3^2} = \sqrt{45} = 3\sqrt{5}$ so it's $\dfrac{6}{3\sqrt{5}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

b) $\vec{a}.\vec{b} = (-5\vec{i}+2\vec{j}).(\vec{i}-7\vec{j})$

$\vec{a}.\vec{b} = -5-14 = -19$

$|\vec{b}| = \sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$

So the scalar projection of $\vec{a}$ onto $\vec{b}$ is $\dfrac{-19}{5\sqrt{2}} = -\dfrac{19\sqrt{2}}{10}$

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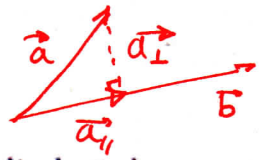


31 For $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + \underset{\sim}{j}$, find:

(a) the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$

a) $\vec{a}\cdot\vec{b} = 8-5 = 3$

$|\vec{b}| = \sqrt{4^2+1^2} = \sqrt{17}$

So the scalar projection is $\frac{3}{\sqrt{17}}$

$\frac{\vec{b}}{|\vec{b}|} = \frac{4\vec{i}+\vec{j}}{\sqrt{17}}$

So the vector projection of $\vec{a}$ onto $\vec{b}$ is $= \frac{3}{\sqrt{17}}\left[\frac{4\vec{i}+\vec{j}}{\sqrt{17}}\right]$

$= \frac{3}{17}\left[4\vec{i}+\vec{j}\right]$

(b) the vector projection of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$.

$\vec{a} = \vec{a_{/\!/}} + \vec{a_\perp}$

$\vec{a} = \left[\frac{3}{17}(4\vec{i}+\vec{j})\right] + \vec{a_\perp} = 2\vec{i} - 5\vec{j}$

$\therefore\ \vec{a_\perp} = \left[2 - \frac{12}{17}\right]\vec{i} + \left[-5 - \frac{3}{17}\right]\vec{j}$

$\vec{a_\perp} = \frac{22}{17}\vec{i} - \frac{88}{17}\vec{j}$

$\vec{a_\perp} = \frac{22}{17}\left[\vec{i} - 4\vec{j}\right]$

32 The points A, B and C have coordinates $(2, -5)$, $(5, 9)$ and $(-9, 12)$ respectively.

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ in column vector form.

(b) Find $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{AC}|$.

(c) Show that ΔABC is an isosceles triangle.

(d) Find the coordinates of a point D such that $ABCD$ forms a rhombus.

(e) Find the coordinates of the point of intersection of the diagonals of the rhombus $ABCD$.

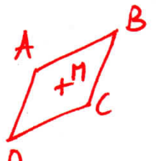
a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB} = -(2\vec{i}-5\vec{j}) + (5\vec{i}+9\vec{j}) = 3\vec{i} + 14\vec{j}$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -\overrightarrow{OB} + \overrightarrow{OC} = -(5\vec{i}+9\vec{j}) + (-9\vec{i}+12\vec{j}) = -14\vec{i} + 3\vec{j}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\overrightarrow{OA} + \overrightarrow{OC} = -(2\vec{i}-5\vec{j}) + (-9\vec{i}+12\vec{j}) = -11\vec{i} + 17\vec{j}$

b) $|\overrightarrow{AB}| = \sqrt{205} = \sqrt{5\times 41}$ $\quad |\overrightarrow{BC}| = \sqrt{205}$ $\quad \overrightarrow{AC} = \sqrt{11^2+17^2} = \sqrt{410}$

c) $|\overrightarrow{AB}| = |\overrightarrow{BC}| = \sqrt{205}$ $\quad\therefore\ \Delta ABC$ is isosceles.



d) $\overrightarrow{AD} = \overrightarrow{BC}$ so $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$

$\overrightarrow{OD} = [2\vec{i}-5\vec{j}] + [-14\vec{i}+3\vec{j}] = -12\vec{i} - 2\vec{j}$ $\quad D(-12, 2)$

e) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = [2\vec{i}-5\vec{j}] + \frac{1}{2}\overrightarrow{AC} = [2\vec{i}-5\vec{j}] + \frac{1}{2}[-11\vec{i}+17\vec{j}]$

$\overrightarrow{OM} = \left[2 - \frac{11}{2}\right]\vec{i} + \left[-5 + \frac{17}{2}\right]\vec{j} = -3.5\vec{i} + 3.5\vec{j}$

so $M(-3.5, 3.5)$

33 **(a)** If $\underset{\sim}{a} = -4e\underset{\sim}{i} + 2e\underset{\sim}{j}$, $e > 0$ and $|\underset{\sim}{a}|^2 = 40$, find the exact value of e. **(b)** Hence, find $\hat{\underset{\sim}{a}}$.
(c) Find the vector $\underset{\sim}{b}$ that is parallel to $\hat{\underset{\sim}{a}}$ with $|\underset{\sim}{b}| = 10$.
(d) If $\underset{\sim}{c} = 4f\underset{\sim}{i} - 3f\underset{\sim}{j}$, $f > 0$ and $|\underset{\sim}{c}|^2 = 250$, find the exact value of f. **(e)** Hence find $\hat{\underset{\sim}{c}}$.
(f) Find the vector $\underset{\sim}{d}$ in the direction of $\hat{\underset{\sim}{c}}$ where $|\underset{\sim}{d}|^2 = 20$. **(g)** Find $\underset{\sim}{b} - \underset{\sim}{d}$.

a) $|\vec{a}| = \sqrt{(4e)^2 + (2e)^2} = \sqrt{16e^2 + 4e^2} = \sqrt{20e^2}$ so $20e^2 = 40$ $e^2 = 2$ $e = \sqrt{2}$

b) $\vec{a} = 2\sqrt{2}\left[-2\vec{i} + \vec{j}\right]$ $|\vec{a}| = \sqrt{40} = 2\sqrt{10}$

$\hat{\underset{\sim}{a}} = \frac{2\sqrt{2}}{2\sqrt{10}}\left[-2\vec{i} + \vec{j}\right] = \frac{1}{\sqrt{5}}\left[-2\vec{i} + \vec{j}\right] = \frac{\sqrt{5}}{5}\left[-2\vec{i} + \vec{j}\right]$

c) $\vec{b} = |\vec{b}| \times \hat{a} = 10 \times \frac{\sqrt{5}}{5}\left[-2\vec{i} + \vec{j}\right] = 2\sqrt{5}\left[-2\vec{i} + \vec{j}\right]$

d) $|\vec{c}|^2 = (4f)^2 + (3f)^2 = 25f^2 = 250$ so $f^2 = 10$ $f = \sqrt{10}$

e) $\vec{c} = \sqrt{10}\left[4\vec{i} - 3\vec{j}\right]$ $|\vec{c}| = \sqrt{250} = 5\sqrt{10}$

$\therefore \hat{\underset{\sim}{c}} = \frac{\sqrt{10}}{5\sqrt{10}}\left[4\vec{i} - 3\vec{j}\right] = \frac{1}{5}\left[4\vec{i} - 3\vec{j}\right]$

f) $\vec{d} = 4k\vec{i} - 3k\vec{j}$ $|\vec{d}|^2 = 25k^2$ and $|d|^2 = 20$

so $k^2 = \frac{20}{25} = \frac{4}{5}$ $k = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\vec{d} = \frac{2\sqrt{5}}{5}\left[4\vec{i} - 3\vec{j}\right]$

g) $\vec{b} - \vec{d} = 2\sqrt{5}\left(-2\vec{i} + \vec{j}\right) - \frac{2\sqrt{5}}{5}\left(4\vec{i} - 3\vec{j}\right)$

$\vec{b} - \vec{d} = \left[-4\sqrt{5} - \frac{8\sqrt{5}}{5}\right]\vec{i} + \left[2\sqrt{5} + \frac{6\sqrt{5}}{5}\right]\vec{j}$

$\vec{b} - \vec{d} = -4\sqrt{5}\left[+1 + \frac{2}{5}\right]\vec{i} + 2\sqrt{5}\left[1 + \frac{3}{5}\right]\vec{j}$

$\vec{b} - \vec{d} = -\frac{28\sqrt{5}}{5}\vec{i} + \frac{16\sqrt{5}}{5}\vec{j} = \frac{4\sqrt{5}}{5}\left[-7\vec{i} + 4\vec{j}\right]$

34 Consider two vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} - \underset{\sim}{j}$.

(a) Find the scalar projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

(b) Find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(c) Find the vector projection of $\underset{\sim}{a}$ perpendicular to the direction of $\underset{\sim}{b}$.

(d) Hence, express the vector $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ in terms of projections parallel to and perpendicular to $\underset{\sim}{b} = -3\underset{\sim}{i} - \underset{\sim}{j}$.

a) $\vec{a}\cdot\vec{b} = (2\vec{i}-5\vec{j})\cdot(-3\vec{i}-\vec{j}) = -6+5 = -1$

$|\vec{b}| = \sqrt{3^2+1^2} = \sqrt{10}$ so the scalar projection of $\vec{a}$ on $\vec{b}$ is $-1/\sqrt{10}$

b) $\dfrac{\vec{b}}{|\vec{b}|} = \dfrac{-3\vec{i}-\vec{j}}{\sqrt{10}}$ so the vector projection of $\vec{a}$ onto $\vec{b}$ is $\dfrac{-1}{\sqrt{10}}\left[\dfrac{-3\vec{i}-\vec{j}}{\sqrt{10}}\right]$

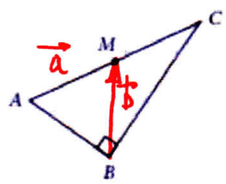
which is $\dfrac{1}{10}[3\vec{i}+\vec{j}]$

c) $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$ so $\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = [2\vec{i}-5\vec{j}] - \dfrac{1}{10}[3\vec{i}+\vec{j}]$

$\vec{a}_{\perp} = \dfrac{17}{10}\vec{i} - \dfrac{51}{10}\vec{j} = \dfrac{17}{10}[\vec{i}-3\vec{j}]$

d) $$\vec{a} = \underbrace{\frac{1}{10}[3\vec{i}+\vec{j}]}_{\parallel \text{ to } \vec{b}} + \underbrace{\frac{17}{10}[\vec{i}-3\vec{j}]}_{\perp \text{ to } \vec{b}}$$

35 ΔABC is right-angled with M being the midpoint of the hypotenuse AC, as shown. Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



(a) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(b) Prove that M is equidistant from the three vertices of ΔABC.

a) $\vec{AB} = \vec{AM} + \vec{MB} = \vec{a} - \vec{BM} = \vec{a} - \vec{b}$

$\vec{BC} = \vec{BM} + \vec{MC} = \vec{b} + \vec{AM} = \vec{b} + \vec{a}$

b) $|\vec{MC}| = |\vec{AM}| = |\vec{a}|$ as M is the midpoint of $\overline{AC}$

$|\vec{MB}|^2 = \vec{MB}\cdot\vec{MB} = (\vec{MA}+\vec{AB})\cdot(\vec{MC}+\vec{CB}) = \vec{MA}\cdot\vec{MC} + \vec{MA}\cdot\vec{CB} + \vec{AB}\cdot\vec{MC} + \underbrace{\vec{AB}\cdot\vec{CB}}_{=0}$

$|\vec{MB}|^2 = -\vec{a}\cdot\vec{a} - \vec{a}\cdot(-\vec{b}-\vec{a}) + (\vec{a}-\vec{b})\vec{a} + 0$ as $\vec{AB}$ and $\vec{CB}$ are perpendicular

$|\vec{MB}|^2 = \cancel{-|\vec{a}|^2} + \vec{a}\cdot\vec{b} \cancel{+|\vec{a}|^2} + |\vec{a}|^2 - \vec{b}\cdot\vec{a}$

$|\vec{MB}|^2 = |\vec{a}|^2$ so $|\vec{MB}| = |\vec{MC}| = |\vec{AM}|$ i.e. M equidistant from A, B and C

36 *OABC* is a parallelogram where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. *M* and *N* are the midpoints of $\overline{AB}$ and $\overline{BC}$ respectively.

(a) Draw a diagram of parallelogram *OABC*, showing the given vectors and midpoints.

(b) Find the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ and show them on your diagram.

(c) Hence find the vector $\overrightarrow{MN}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

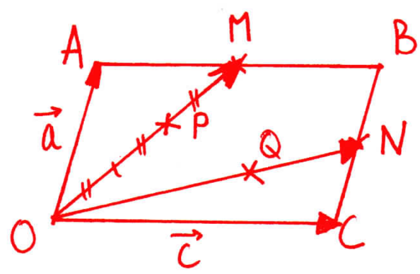
(d) Find vector $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ and show this on your diagram.

(e) *P* is a point on $\overrightarrow{OM}$ such that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OM}$. Find the vector $\overrightarrow{OP}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(f) Q is a point on $\overrightarrow{ON}$ such that $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{ON}$. Find the vector $\overrightarrow{OQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(g) Show that vector $\overrightarrow{MN}$ is parallel to and half the magnitude of $\overrightarrow{AC}$.

(h) Find vectors $\overrightarrow{AP}$, $\overrightarrow{PQ}$ and $\overrightarrow{QC}$, and hence prove that the diagonal $\overline{AC}$ is trisected at *P* and *Q*.



b) $\vec{OM} = \vec{OA} + \vec{AM} = \vec{a} + \frac{1}{2}\vec{c}$.

$\vec{ON} = \vec{OC} + \vec{CN} = \vec{c} + \frac{1}{2}\vec{a}$

c) $\vec{MN} = \vec{MO} + \vec{ON} = -\left(\vec{a} + \frac{1}{2}\vec{c}\right) + \left(\vec{c} + \frac{1}{2}\vec{a}\right)$

$\vec{MN} = \frac{1}{2}\vec{c} - \frac{1}{2}\vec{a} = \frac{1}{2}(\vec{c} - \vec{a})$

d) $\vec{AC} = \vec{AO} + \vec{OC} = -\vec{a} + \vec{c} = \vec{c} - \vec{a}$

e) $\vec{OP} = \frac{2}{3}\vec{OM} = \frac{2}{3}\left[\vec{a} + \frac{1}{2}\vec{c}\right] = \frac{2}{3}\vec{a} + \frac{1}{3}\vec{c} = \frac{1}{3}\left[2\vec{a} + \vec{c}\right]$

f) $\vec{OQ} = \frac{2}{3}\vec{ON} = \frac{2}{3}\left[\vec{c} + \frac{1}{2}\vec{a}\right] = \frac{1}{3}\left[2\vec{c} + \vec{a}\right]$

g) $\vec{MN} = \frac{1}{2}(\vec{c} - \vec{a})$ whereas $\vec{AC} = \vec{AO} + \vec{OC} = -\vec{OA} + \vec{c} = \vec{c} - \vec{a}$

Hence $\vec{MN} = \frac{1}{2}\vec{AC}$

h) $\vec{AP} = \vec{AO} + \vec{OP} = -\vec{a} + \frac{1}{3}\left[2\vec{a} + \vec{c}\right] = -\frac{1}{3}\vec{a} + \frac{1}{3}\vec{c} = \frac{1}{3}\left[\vec{c} - \vec{a}\right]$

$\vec{PQ} = \vec{PO} + \vec{OQ} = -\vec{OP} + \vec{OQ} = -\frac{1}{3}\left[2\vec{a} + \vec{c}\right] + \frac{1}{3}\left[2\vec{c} + \vec{a}\right]$

$\therefore \vec{PQ} = -\frac{1}{3}\vec{a} + \frac{1}{3}\vec{c} = \frac{1}{3}(\vec{c} - \vec{a})$

$\vec{QC} = \vec{QO} + \vec{OC} = -\vec{OQ} + \vec{OC} = -\frac{1}{3}\left[2\vec{c} + \vec{a}\right] + \vec{c} = \frac{1}{3}\left[\vec{c} - \vec{a}\right]$

$\therefore \vec{AP} = \vec{PQ} = \vec{QC}$ And as A, P, Q, C are colinear that means the diagonal AC is trisected at P and Q.