APPLICATIONS INVOLVING INTEGRALS

1 A cube of ice has an edge length of 10 cm. It melts so that its volume decreases at a constant rate and the block remains a cube. If the edge length measures 5 cm after 70 minutes, find:

(a) the rate at which the volume decreases **(b)** the volume at any time t.

2 A machine manufactures items at a variable rate given by $\frac{dQ}{dt} = 2t + 1$, $t \geq 0$, where Q is the number of items manufactured in a time t minutes.

(a) At what rate is the machine working: **(i)** initially **(ii)** after 10 minutes?

(b) What is the total number of items manufactured in the first 10 minutes?

3 The sluice gates of a dam are operated by an automatic program that controls the flow of water out of the dam. The program is set so that t hours after 7 am the flow of water will be given by $\frac{dV}{dt} = 500 - 15t^2 + t^3$ megalitres (ML) per hour.

(a) If no water flows from the dam before 7 am, calculate:

(i) the flow of the water at 9 am

(ii) the total volume of water released between 7 am and 9 am

(b) **(i)** Sketch $\frac{dV}{dt} = 500 - 15t^2 + t^3$ for $0 \leq t \leq 10$.

(ii) When does the flow of water stop?

(iii) If the sluice gates close at the moment when $\frac{dV}{dt} = 0$, how much water has been released altogether?

5 A body starts from O and moves in a straight line. At any time t its velocity is given by $\dot{x} = 6t - 4$. Indicate whether each statement below is correct or incorrect.

(a) $x = 3t^2 - 4t + C$ **(b)** $x = 3t^2 - 4t$ **(c)** $\ddot{x} = 3t^2 - 4t$ **(d)** $\ddot{x} = 6$

6 A body starts from O and moves in a straight line. At any time t, its velocity is $t^2 - 4t^3$. Find, in terms of t:

(a) the displacement x **(b)** the acceleration.

7 The velocity v m s^{-1} at time t seconds ($t \geq 0$) of a body moving in a straight line is given by $v = 6t^2 + 6t - 12$. Its initial displacement is 7 m from O. Find:

(a) the displacement and acceleration at any time t

(b) the acceleration when the velocity is zero **(c)** the initial velocity and acceleration.

9 A body is projected vertically upwards with an initial velocity of $30\,\text{m}\,\text{s}^{-1}$. It rises with a deceleration of $10\,\text{m}\,\text{s}^{-2}$. Find:

(a) its velocity at any time t

(b) its height h m above the point of projection at any time t

(c) the greatest height reached

(d) the time taken to return to the point of projection.

14 Two cars A and B travel along a straight road in the same direction. Their respective distances x km from a fixed point O at any time t hours are given by the following rules:

A: $x = 50t - 20t^2$ B: $x = 80t^2 + 20t$

(a) Calculate each car's speed at the point O.

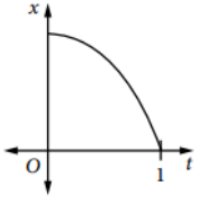
(b) At what time are the cars travelling at the same speed?

(c) Both cars reach a point Q at the same time. Calculate the distance from O to Q.

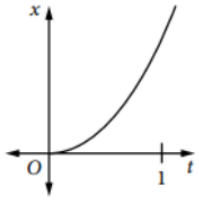
(d) A third car, travelling at uniform speed, is 2 km ahead of A and B when they pass the point O. If this car arrives at Q at the same time as A and B, find a rule connecting x and t for it.

16 A particle is moving so that, for $0 < t < 1$, its velocity is positive and its acceleration is negative. Which graph could represent the displacement function of this particle?

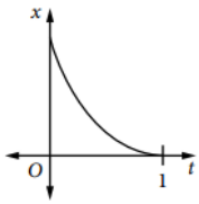
A



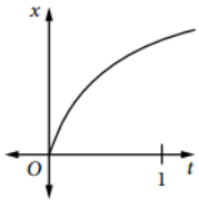
B



C



D



21 A particle moves in a straight line so that at time t its displacement from a fixed origin is x and its velocity is v.

(a) If its acceleration is $2\cos t$, and $v = 1$ and $x = 0$ when $t = 0$, find x in terms of t.

(b) If its acceleration is $-3e^{-t}$ and $v = 0$ when $t = 0$, find the time at which $v = -2$.

23 The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = 12\cos 2t$. Initially $v = 0$ and $x = 6$. Find its velocity v and displacement x at any time t seconds and sketch the graph of the displacement. How many times does the particle change direction in the first 10 seconds?