PROJECTION OF VECTORS

1 For each of the following pairs of vectors, find the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(a) $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ **(c)** $\underset{\sim}{a} = 8\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 8\underset{\sim}{j}$

2 For each of the following pairs of vectors, find the vector projections of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(a) $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ **(c)** $\underset{\sim}{a} = 8\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 6\underset{\sim}{j}$

3 For each of the following pairs of vectors, find the vector projections of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$.

(a) $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ **(c)** $\underset{\sim}{a} = 8\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 6\underset{\sim}{j}$

4 For the following vectors, find the scalar projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$.

(a) $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$

(b) $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

6 For $\underset{\sim}{a} = -2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$, the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ is:

A $\frac{\sqrt{2}}{2}$ **B** $\frac{-\sqrt{2}}{2}$ **C** $\frac{-2\sqrt{13}}{13}$ **D** $\frac{2\sqrt{13}}{13}$

7 The vector projection of $3\underset{\sim}{i} + 2\underset{\sim}{j}$ onto $-\underset{\sim}{i} + 2\underset{\sim}{j}$ is $\frac{1}{5}\left(-\underset{\sim}{i} + 2\underset{\sim}{j}\right)$. What is the vector projection of $3\underset{\sim}{i} + 2\underset{\sim}{j}$ perpendicular to $-\underset{\sim}{i} + 2\underset{\sim}{j}$?

A $\frac{8}{5}\left(3\underset{\sim}{i} + 2\underset{\sim}{j}\right)$ **B** $\frac{8}{5}\left(-\underset{\sim}{i} + 2\underset{\sim}{j}\right)$ **C** $\frac{8}{5}\left(2\underset{\sim}{i} + \underset{\sim}{j}\right)$ **D** $2\underset{\sim}{i} + \underset{\sim}{j}$

9 Consider two vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$.

(a) Find the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(b) Find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(c) Find the vector projection of $\underset{\sim}{a}$ perpendicular to the direction of $\underset{\sim}{b}$.

(d) Hence, express the vector $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ in terms of projections onto and perpendicular to $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$.

11 $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are unit vectors in the Cartesian plane.

(a) Show that $\underset{\sim}{a} = \cos\alpha\,\underset{\sim}{i} + \sin\alpha\,\underset{\sim}{j}$.

(b) Derive similar expressions for $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

(c) Find $\underset{\sim}{a} \bullet \underset{\sim}{b}$ and $\underset{\sim}{a} \bullet \underset{\sim}{c}$.

(d) Hence deduce the compound angle formulas
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ and
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.