INTRODUCTION TO VECTORS

Scalar quantities and Vector quantities

A **scalar** quantity is one that is completely specified by its magnitude (size) and, if appropriate, its unit of measurement. Examples of scalar quantities include mass, length, speed, temperature and time.

Scalar quantities can have negative values, such as temperature.

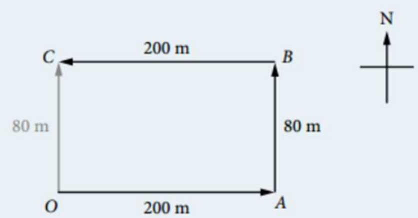
A **vector** quantity is one that is completely described by its magnitude and direction (and if appropriate, its unit of measurement). Examples of vector quantities include:

- displacement (change in position of an object)
- velocity (time rate of change of displacement or position)
- acceleration (time rate of change of velocity)
- force (a push or a pull that can affect the motion of an object)
- weight (the gravitational force acting on an object, which is not the same as the mass of an object, which is a scalar)
- momentum (the product of an object's mass and velocity)

Example 1

Sarah walks from point O to point C, via points A and B.

(a) What distance has she walked?
(b) What is her position relative to her starting point?



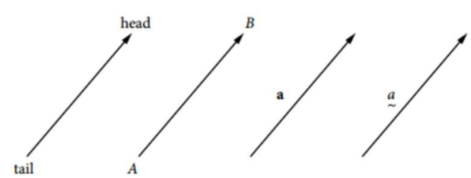
Solution

(a) Sarah walks a total distance of 480 m.
(b) Sarah is 80 m directly north of her starting point at O. This is her displacement from O.

Vectors and vector notation

A vector quantity can be represented by a directed straight line segment with an arrowhead on the line segment indicating its direction and the length of the line indicating its magnitude.

The beginning of the vector is called its tail and the end its head.



The vector with a tail at point A and head at point B is noted $\overrightarrow{AB}$, or $\vec{a}$

Two alternative notations are also used:

- in bold letters, such as $\boldsymbol{a}$ or $\boldsymbol{AB}$ (particularly in Physics textbooks)
- with a tilde underneath $\underset{\sim}{a}$.

In handwritten text, use either the arrow or tilde underneath notations.

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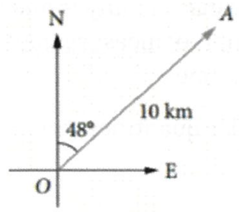
Magnitude of a vector

The magnitude of a vector is a measure of its size or length and is a scalar quantity.

The magnitude of vector $\overrightarrow{AB}$ is written $|\overrightarrow{AB}|$, the magnitude of vector $\underset{\sim}{a}$ is written $|\underset{\sim}{a}|$, and the magnitude of vector **a** is written $|\mathbf{a}|$.

The diagram at right shows a vector $\overrightarrow{OA}$ of magnitude 10 km in a direction N48°E.

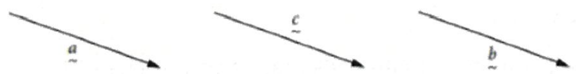


Vector algebra

Equality of vectors

Two vectors are equal if and only if they have the same magnitude and the same direction, regardless of their positions.

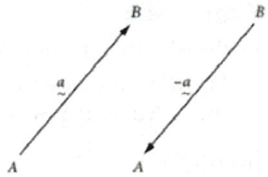
In the example below, $\underset{\sim}{a} = \underset{\sim}{b} = \underset{\sim}{c}$.



Negative of a vector

The negative of vector $\overrightarrow{AB} = \underset{\sim}{a}$ is the vector $-\overrightarrow{AB} = \overrightarrow{BA} = -\underset{\sim}{a}$.

$-\underset{\sim}{a}$ has the same magnitude as $\underset{\sim}{a}$ but with opposite direction. $\overrightarrow{BA}$ is the vector drawn from B to A, whereas $\overrightarrow{AB}$ is the vector drawn from A to B. Therefore, $\overrightarrow{BA} = -\overrightarrow{AB}$ and the direction is reversed.



The zero vector

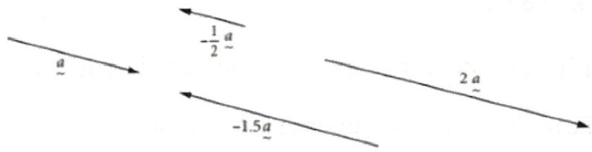
The zero vector, denoted $\underset{\sim}{0}$, is a vector of zero magnitude. Its direction cannot be defined. Adding a vector to its negative will produce the zero vector.

$\underset{\sim}{a} + (-\underset{\sim}{a}) = \underset{\sim}{0}$

Scalar multiplication of vectors

Multiplying a vector by a scalar (a number) k, where $k \in R$, the set of real numbers, $k \neq 0$, results in a vector with k times the magnitude and parallel to the original vector.

- If $k > 0$, then $k\underset{\sim}{a}$ has the same direction as $\underset{\sim}{a}$, but has k times the magnitude.
- If $k = 0$, then $k\underset{\sim}{a} = \underset{\sim}{0}$.
- If $k < 0$, then $k\underset{\sim}{a}$ is in the opposite direction to $\underset{\sim}{a}$ and has k times the magnitude.



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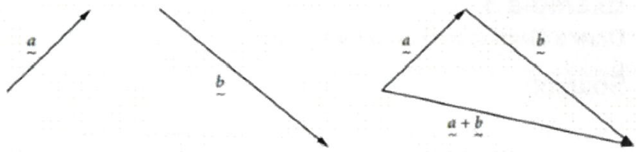
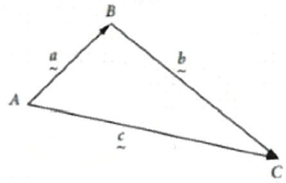
Addition of vectors

The triangle rule for the addition of vectors

To add two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, the vectors are placed with the head of $\underset{\sim}{a}$ at the tail of $\underset{\sim}{b}$. The resultant vector or vector sum is a vector joining the tail of $\underset{\sim}{a}$ to the head of $\underset{\sim}{b}$. This vector addition forms a triangle, hence the **triangle rule** for vector addition, sometimes called the head-to-tail rule.



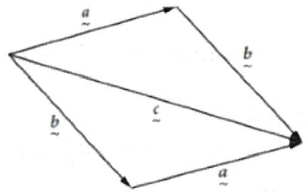
Also, for ΔABC, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ or $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$.



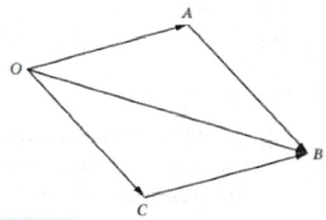
The parallelogram rule for the addition of vectors

If two vectors to be added have the same initial point, they can be added by drawing a parallelogram rather than moving one vector to the head of the other vector. The diagonal of the parallelogram, drawn from the initial point to the final point, represents the vector sum. This works because adding $\underset{\sim}{b}$ to $\underset{\sim}{a}$ is the same as adding $\underset{\sim}{a}$ to $\underset{\sim}{b}$.

The diagram shows that $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{b}+\underset{\sim}{a}=\underset{\sim}{c}$ and this rule is known as the **parallelogram rule** for addition of vectors.
The parallelogram rule for addition of vectors illustrates that vector addition is commutative.



For parallelogram $OABC$, you can also write $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$ and $\overrightarrow{OC}+\overrightarrow{CB}=\overrightarrow{OB}$.



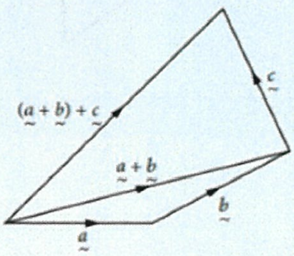
Example 2

Draw diagrams to show:

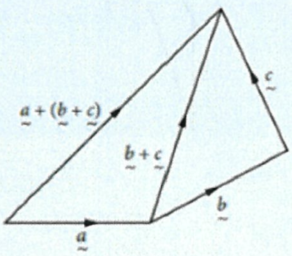
(a) $(\underset{\sim}{a}+\underset{\sim}{b})+\underset{\sim}{c}$ (b) $\underset{\sim}{a}+(\underset{\sim}{b}+\underset{\sim}{c})$

Solution

(a)



(b)



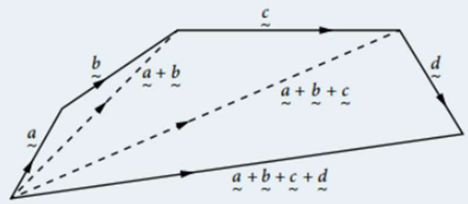
$(\underset{\sim}{a}+\underset{\sim}{b})+\underset{\sim}{c}=\underset{\sim}{a}+(\underset{\sim}{b}+\underset{\sim}{c})=\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}$

Vector addition is also associative. If three vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are added, then $\underset{\sim}{a}+(\underset{\sim}{b}+\underset{\sim}{c})=(\underset{\sim}{a}+\underset{\sim}{b})+\underset{\sim}{c}=\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}$. This method can be extended to any number of vectors (sometimes called the **polygon rule** for vector addition).

Example 3

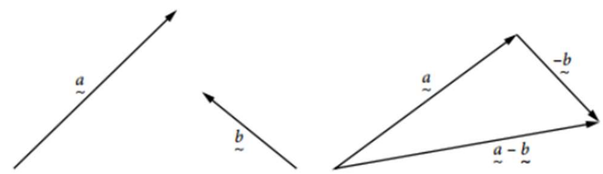
Draw a diagram to show $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}+\underset{\sim}{d}$.

Solution



Subtraction of vectors

Subtraction of a vector is defined as addition of its negative. That is, to subtract $\underset{\sim}{b}$ from $\underset{\sim}{a}$, add $-\underset{\sim}{b}$ to $\underset{\sim}{a}$, and so $\underset{\sim}{a}-\underset{\sim}{b}=\underset{\sim}{a}+(-\underset{\sim}{b})$.

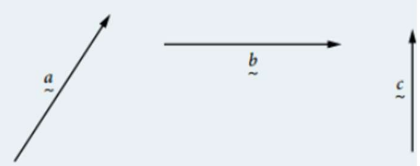


If a vector is subtracted from itself, the result is the zero vector: $\underset{\sim}{a}-\underset{\sim}{a}=\underset{\sim}{0}$.

Example 4

Given the three vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, as shown, construct the following:

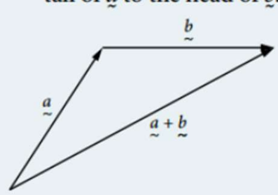
(a) $\underset{\sim}{a}+\underset{\sim}{b}$
(b) $\underset{\sim}{c}+\underset{\sim}{a}$
(c) $\underset{\sim}{b}-\underset{\sim}{c}$



Solution

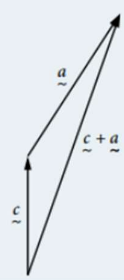
(a) To add two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, the vectors are placed with the head of $\underset{\sim}{a}$ at the tail of $\underset{\sim}{b}$.

The resultant vector or vector sum is a vector joining the tail of $\underset{\sim}{a}$ to the head of $\underset{\sim}{b}$.



(b) To add two vectors $\underset{\sim}{c}$ and $\underset{\sim}{a}$, the vectors are placed with the head of $\underset{\sim}{c}$ at the tail of $\underset{\sim}{a}$.

The resultant vector or vector sum is a vector joining the tail of $\underset{\sim}{c}$ to the head of $\underset{\sim}{a}$.



(c) To subtract vector $\underset{\sim}{c}$ from vector $\underset{\sim}{b}$, the vector $-\underset{\sim}{c}$ is placed with the head of $\underset{\sim}{b}$ at the tail of $-\underset{\sim}{c}$.

The resultant vector or vector sum is a vector joining the tail of $\underset{\sim}{b}$ to the head of $-\underset{\sim}{c}$.