DERIVATIVE OF $f(x) = e^x$

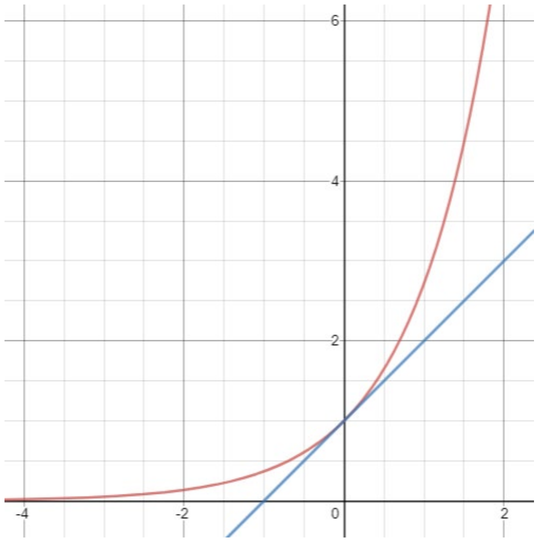
Exponential functions are of the form $f(x) = a^x$, with a a positive constant.

All these functions pass through the point $(0,1)$, as when $x = 0$, $f(0) = a^0 = 1$

The function $f(x) = e^x$ is defined as being the exponential function for which the slope of the tangent at the point $(0,1)$ is 1 i.e.:

$$\lim_{h\to 0}\left(\frac{e^h - e^0}{h}\right) = 1$$

which can also be noted: $\lim_{h\to 0}\left(\frac{e^h-1}{h}\right) = 1$



The value of the number "e" is approximately 2.71828182845…(it never ends, does not repeat, is irrational (i.e. cannot be written as a fraction) and transcendental (i.e. cannot be a solution of a polynomial equation with rational coefficients).

It was named "e" after mathematician Leonhard Euler who studied it extensively around beginning of 18th century.

This function $f(x) = e^x$ is called "*the **natural** exponential function*" as of all exponential functions $f(x) = a^x$, with a a positive constant, it is the only one whose gradient at the point $(0,1)$ is 1.

To find the derivative of $f(x) = e^x$, we use the first principle of differentiation:

$$f'(x) = \lim_{h\to 0}\left(\frac{e^{x+h} - e^x}{h}\right) = \lim_{h\to 0}\left(\frac{e^x e^h - e^x}{h}\right) = \lim_{h\to 0}\left(\frac{e^x(e^h - 1)}{h}\right) = \lim_{h\to 0} e^x\left(\frac{e^h - 1}{h}\right)$$

We can replace 1 by e^0 as: $1 = e^0$

$$f'(x) = e^x \times \lim_{h\to 0}\left(\frac{e^h - e^0}{h}\right)$$

But $\left(\frac{e^h-e^0}{h}\right)$ is the slope of the function $f(x) = e^x$ at $x = 0$, which is equal to 1 (by definition of "e") and therefore: $\lim_{h\to 0}\left(\frac{e^h-e^0}{h}\right) = 1$

Therefore: $f'(x) = e^x \times 1$

$$f'(x) = e^x$$

So the derivative of $f(x) = e^x$ is itself, i.e. $f'(x) = e^x$

This is the only function which is equal to its derivative.

DERIVATIVE OF $f(x) = e^{kx}$ where k is a constant

$$f(x) = e^{kx} = g[h(x)]$$

We deal with this derivative as it is a composition of functions.

$$g(X) = e^X \qquad\qquad h(x) = kx$$

$$g'(X) = e^X \qquad\qquad h'(x) = k$$

Therefore, using the chain rule:

$$f'(x) = g'[h(x)] \times h'(x)$$

$$f'(x) = e^{kx} \times k$$

$$f'(x) = k\, e^{kx}$$

Example: if $f(x) = e^{-3x}$ then $f'(x) = -3\, e^{-3x}$