DEFINITE INTEGRALS INVOLVING EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1 Find the value of:

(a) $\int_{-1}^{1} e^{x}\,dx$ **(b)** $\int_{0}^{2} e^{2x}\,dx$ **(c)** $\int_{-1}^{3} e^{-\frac{x}{2}}\,dx$ **(d)** $\int_{0}^{1} e^{1.5t}\,dt$

2 Indicate whether each statement below is a correct or incorrect step in the evaluation of $I=\int_{-1}^{1}\left(e^{x}-e^{-x}\right)^{2}dx$.

(a) $I=\int_{-1}^{1}\left(e^{2x}-2+e^{-2x}\right)dx$ **(b)** $I=\left[\frac{e^{2x}}{2}-2x-\frac{e^{-2x}}{2}\right]_{-1}^{1}$

(c) $I=\left[e^{2x}-4x-e^{-2x}\right]_{0}^{1}$ **(d)** $I=\frac{e^{4}-4e^{2}-1}{4e^{2}}$

4 **(a)** Calculate the area bounded by the curve $y = e^x$, the coordinate axes and the line $x = 2$.
(b) Write the equation of the tangent to $y = e^x$ at the point where $x = 2$.
(c) Calculate the area bounded by $y = e^x$, the coordinate axes and the tangent at $x = 2$.
(d) Calculate the area bounded by $y = e^x$, the y-axis and the line $y = e^2$.

6 Calculate the area bounded by the curve $y = e^{0.5x} - e^{-0.5x}$, the x-axis and the line $x = 1$.

7 Evaluate: (a) $\int_2^3 \frac{1}{x-1}dx$ (b) $\int_0^3 \frac{2}{x+3}dx$ (c) $\int_{-2}^0 \frac{dx}{5+2x}$

(e) $\int_2^3 \left(x+\frac{1}{x-1}\right)dx$ (f) $\int_1^2 \left(x-\frac{1}{x^2}\right)^2 dx$ (g) $\int_1^3 \left(e^x+\frac{1}{x}\right)dx$

10 Find the area of the region enclosed by the curve $y = \frac{x}{x^2+1}$, the x-axis and the ordinates $x = 2$ and $x = 4$.

14 $\int_0^1 \frac{e^x}{1+e^x}\,dx = \log_e c$. Find the value of c.

16 a) Differentiate $f(x) = x \ln x$

b) Find the area enclosed by the function $y = \ln x$ and the lines $y = 0$ and $x = a$ where $a > 1$

17 **(a)** Find $\frac{d}{dx}\left(\log_e(\cos x)\right)$.

(b) Find the area enclosed by the curve $y = \tan x$, the x-axis and the ordinate $x = \frac{\pi}{3}$.