SIMPLE TRIGONOMETRIC EQUATIONS

1 Solve for values of θ and x between 0 and 2π inclusive:

(a) $\sin\theta = \frac{\sqrt{3}}{2}$ (b) $\tan x = -1$ (c) $\cos x = -0.5$ (d) $\sqrt{3}\tan\theta = 1$

a) $\sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$ The general solution of the equation $\sin\theta = x$ is $\theta = (-1)^n \times \sin^{-1}x + n\pi$ so $\theta = (-1)^n \times \frac{\pi}{3} + n\pi$

For $n=0$ $\theta = \pi/3$ $n=1$ $\theta = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

There are no other solutions

b) $\tan x = -1 = \tan\frac{3\pi}{4}$

The general solution of the equation $\tan\theta = x$ is $\theta = \tan^{-1}x + n\pi$

So $x = \frac{3\pi}{4} + n\pi$ For $n=0$ $x = \frac{3\pi}{4}$ For $n=1$ $x = \frac{7\pi}{4}$

no other solutions

c) $\cos x = -0.5 = \cos\frac{2\pi}{3}$

The general solution of the equation $\cos\theta = x$ is $\theta = \pm\cos^{-1}x + n\times 2\pi$

So $x = \pm\frac{2\pi}{3} + n\times 2\pi$ For $n=0$ $x = \frac{2\pi}{3}$ For $n=1$ $x = \frac{4\pi}{3}$

no other solutions

d) $\sqrt{3}\tan\theta = 1 \iff \tan\theta = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \tan\frac{\pi}{6}$

So $\theta = \frac{\pi}{6} + n\pi$

For $n=0$ $\theta = \frac{\pi}{6}$

For $n=1$ $\theta = \frac{7\pi}{6}$

no other solutions

SIMPLE TRIGONOMETRIC EQUATIONS

1 Solve for values of θ and x between 0 and 2π inclusive:

(e) $\sin 2\theta = -\frac{1}{2}$ **(f)** $\operatorname{cosec}\theta = -2$ **(g)** $\cot 2x = \sqrt{3}$ **(h)** $\sec 2\theta = \sqrt{2}$

e) $\sin 2\theta = -\frac{1}{2} = \sin\left(\frac{-\pi}{6}\right)$ so $2\theta = (-1)^n\left(\frac{-\pi}{6}\right) + n\pi \Leftrightarrow \theta = (-1)^n\left(\frac{-\pi}{12}\right) + n\frac{\pi}{2}$

For $n=0$ $\theta = -\pi/12$ (outside of $[0, 2\pi]$)

For $n=1$ $\theta = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ For $n=2$ $\theta = -\frac{\pi}{12} + \pi = \frac{11\pi}{12}$

For $n=3$ $\theta = \frac{\pi}{12} + \frac{3\pi}{2} = \frac{19\pi}{12}$ For $n=4$ $\theta = -\frac{\pi}{12} + \frac{4\pi}{2} = \frac{23\pi}{12}$

f) $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = -2$ so $\sin\theta = -\frac{1}{2} = \sin\left(\frac{-\pi}{6}\right)$

General solution is $\theta = (-1)^n\left(\frac{-\pi}{6}\right) + n\pi$

For $n=0$ $\theta = -\pi/6$ (outside of range)

$n=1$ $\theta = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$ $n=2$ $\theta = -\frac{\pi}{6} + 2\pi = \frac{11\pi}{6}$ no other solutions

g) $\cot 2x = \sqrt{3} \Leftrightarrow \tan 2x = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \tan\frac{\pi}{6}$

General solution is $2x = \frac{\pi}{6} + n\pi \Leftrightarrow x = \frac{\pi}{12} + n\frac{\pi}{2}$

$n=0$ $x = \frac{\pi}{12}$ $n=1$ gives $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$

$n=2$ $x = \frac{\pi}{12} + \pi = \frac{13\pi}{12}$ $n=3$ gives $x = \frac{\pi}{12} + \frac{3\pi}{2} = \frac{19\pi}{12}$

h) $\sec 2\theta = \sqrt{2} \Leftrightarrow \frac{1}{\cos 2\theta} = \sqrt{2} \Leftrightarrow \cos 2\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

General solution is $2\theta = \pm\frac{\pi}{4} + 2n\pi \Leftrightarrow \theta = \pm\frac{\pi}{8} + n\pi$

$n=0$ gives $\theta = \pi/8$ $n=1$ gives $\theta = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$

$\theta = -\frac{\pi}{8} + \pi = \frac{7\pi}{8}$

$n=2$ gives $\theta = \pm\frac{\pi}{8} + 2\pi$

$\theta = -\frac{\pi}{8} + 2\pi = \frac{15\pi}{8}$

no others solutions outside of $[0, 2\pi]$

SIMPLE TRIGONOMETRIC EQUATIONS

3 The solution to $\sqrt{2}\sin 2\theta + 1 = 0$ for $0 \le \theta \le 2\pi$ is:

A $\frac{5\pi}{4}, \frac{7\pi}{4}$ B $\frac{5\pi}{8}, \frac{7\pi}{8}$ $\boxed{\text{C}}$ $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ D $\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

$\sqrt{2}\sin 2\theta + 1 = 0 \iff \sin 2\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} = \sin\left(\frac{-\pi}{4}\right)$

General solution is $2\theta = (-1)^n \times \left(\frac{-\pi}{4}\right) + n\pi \iff \theta = (-1)^n\left(\frac{-\pi}{8}\right) + \frac{n\pi}{2}$

$n=0$ gives $\theta = -\frac{\pi}{8}$ (outside of range)

$n=1$ gives $\theta = \frac{\pi}{8} + \frac{\pi}{2} = \frac{5\pi}{8}$

$n=2$ gives $\theta = -\frac{\pi}{8} + \pi = \frac{7\pi}{8}$

$n=3$ gives $\theta = \frac{\pi}{8} + \frac{3\pi}{2} = \frac{13\pi}{8}$

$n=4$ gives $\theta = -\frac{\pi}{8} + 2\pi = \frac{15\pi}{8}$ so $\boxed{C}$

4 Solve for $-\pi \le x \le \pi$: **(a)** $2\cos 2x + 1 = 0 \iff \cos 2x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

General solution is $2x = \pm\left(\frac{2\pi}{3}\right) + 2n\pi \iff x = \pm\left(\frac{\pi}{3}\right) + n\pi$

$n=0$ gives $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$

$n=1$ gives $x = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$ (outside $[-\pi, \pi]$)

$x = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

$n=-1$ gives $x = \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

$x = -\frac{\pi}{3} - \pi = -\frac{4\pi}{3}$ (outside $[-\pi, \pi]$)

So 4 solutions $-\frac{2\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

5 Solve between 0 and 2π inclusive:

(a) $\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ (b) $\tan\left(\theta-\frac{\pi}{3}\right)=-\sqrt{3}$ (c) $\cos\left(2x+\frac{\pi}{3}\right)=\frac{1}{2}$

a) $\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\sin\frac{\pi}{4}$

General solution is $\theta+\frac{\pi}{4}=(-1)^n\frac{\pi}{4}+n\pi \iff \theta=(-1)^n\frac{\pi}{4}-\frac{\pi}{4}+n\pi$

$n=0$ gives $\theta=0$

$n=1$ gives $\theta=-\frac{\pi}{4}-\frac{\pi}{4}+\pi=\frac{\pi}{2}$

$n=2$ gives $\theta=(-1)^2\frac{\pi}{4}-\frac{\pi}{4}+2\pi=\frac{\pi}{4}-\frac{\pi}{4}+2\pi=2\pi$

So 3 solutions $0, \frac{\pi}{2}, 2\pi$

b) $\tan\left(\theta-\frac{\pi}{3}\right)=-\sqrt{3}=\frac{-\sqrt{3}/2}{1/2}=\tan\left(-\frac{\pi}{3}\right)$

General solution is $\theta-\frac{\pi}{3}=-\frac{\pi}{3}+n\pi \iff \theta=-\frac{\pi}{3}+\frac{\pi}{3}+n\pi=n\pi$

$n=0$ gives $\theta=0$

$n=1$ gives $\theta=\pi$

$n=2$ gives $\theta=2\pi$ So 3 solutions $0, \pi, 2\pi$

c) $\cos\left(2x+\frac{\pi}{3}\right)=\frac{1}{2}=\cos\frac{\pi}{3}$

General solutions is $2x+\frac{\pi}{3}=\pm\frac{\pi}{3}+2n\pi \iff 2x=\pm\frac{\pi}{3}-\frac{\pi}{3}+2n\pi$

$\iff x=\pm\frac{\pi}{6}-\frac{\pi}{6}+n\pi$

$n=0$ gives $x=\frac{\pi}{6}-\frac{\pi}{6}=0$ or $x=-\frac{\pi}{6}-\frac{\pi}{6}=-\frac{\pi}{3}$ (outside)

$n=1$ gives $x=\frac{\pi}{6}-\frac{\pi}{6}+\pi=\pi$ or $x=-\frac{\pi}{6}-\frac{\pi}{6}+\pi=\frac{2\pi}{3}$

$n=2$ gives $x=\frac{\pi}{6}-\frac{\pi}{6}+2\pi=2\pi$ or $x=-\frac{\pi}{6}-\frac{\pi}{6}+2\pi=\frac{5\pi}{3}$

So 5 solutions $0, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

(values for other integers are outside of the interval $[0, 2\pi]$)

5 Solve between 0 and 2π inclusive:

(e) $\tan\left(2\theta - \frac{\pi}{4}\right) + 1 = 0$ **(f)** $2\cos\left(2x - \frac{\pi}{3}\right) = \sqrt{3}$

e) $\tan\left(2\theta - \frac{\pi}{4}\right) = -1 = \tan\left(\frac{3\pi}{4}\right)$

General solution is $2\theta - \frac{\pi}{4} = \frac{3\pi}{4} + n\pi \iff 2\theta = \frac{3\pi}{4} + \frac{\pi}{4} + n\pi$

$\iff 2\theta = \pi + n\pi \iff \boxed{\theta = \frac{\pi}{2} + n\frac{\pi}{2}}$

$n = 0$ gives $\theta = \frac{\pi}{2}$ $\quad$ $n = -1$ gives $\theta = 0$

$n = 1$ gives $\theta = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ $\quad$ $n = 2$ gives $\theta = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$

$n = 3$ gives $\theta = \frac{\pi}{2} + \frac{3\pi}{2} = 2\pi$

So 5 solutions: $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

f) $2\cos\left(2x - \frac{\pi}{3}\right) = \sqrt{3} \iff \cos\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

General solution is $2x - \frac{\pi}{3} = \pm\frac{\pi}{6} + 2n\pi$

$\iff 2x = \pm\frac{\pi}{6} + \frac{\pi}{3} + 2n\pi \iff \boxed{x = \pm\frac{\pi}{12} + \frac{\pi}{6} + n\pi}$

$n = 0$ gives $x = \frac{\pi}{12} + \frac{\pi}{6} = \frac{3\pi}{12} = \frac{\pi}{4}$ and $x = -\frac{\pi}{12} + \frac{\pi}{6} = \frac{\pi}{12}$

$n = 1$ gives $x = \frac{\pi}{12} + \frac{\pi}{6} + \pi = \frac{5\pi}{4}$ and $x = -\frac{\pi}{12} + \frac{\pi}{16} + \pi = \frac{13\pi}{12}$

other solutions are outside of the interval $[0, 2\pi]$

So 4 solutions: $\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{13\pi}{12}$

SIMPLE TRIGONOMETRIC EQUATIONS

7 If $0 \le x \le 2\pi$, the solution to $\sin x \le \frac{\sqrt{3}}{2}$ is:

A $x \le \frac{\pi}{3}$ **B** $x \le \frac{\pi}{3}$ or $x \ge \frac{2\pi}{3}$ **C** $0 \le x \le \frac{\pi}{3}$ or $x \ge \frac{2\pi}{3}$ $\boxed{\textbf{D}}$ $0 \le x \le \frac{\pi}{3}$ or $\frac{2\pi}{3} \le x \le 2\pi$

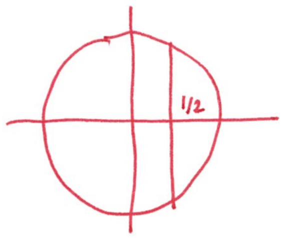
$\sin x \leqslant \frac{\sqrt{3}}{2} \iff \sin x \leqslant \sin \frac{\pi}{3}$

so $0 \leqslant x \leqslant \frac{\pi}{3}$ or $\frac{2\pi}{3} \leqslant x \leqslant 2\pi$ Response $\boxed{D}$

8 If $0 \le x \le 2\pi$, solve: **(a)** $\sin x \ge \frac{1}{2}$ **(b)** $\cos x < \frac{1}{2}$

a) $\sin x \geqslant \sin \frac{\pi}{6}$ $\frac{\pi}{6} \leqslant x \leqslant \frac{5\pi}{6}$

b) $\cos x < \frac{1}{2} \iff \cos x < \cos \frac{\pi}{3}$



$\frac{\pi}{3} \leqslant x \leqslant \frac{5\pi}{3}$