SIMULTANEOUS LINEAR INEQUALITIES

Two linear equations may intersect at a point. The intersection of two linear inequalities is the region common to the two inequalities. This is the region where both inequalities hold simultaneously.

Example 9

(a) Sketch the region given by $y \geq x$. **(b)** Sketch the region given by $x + y < 2$.

(c) Sketch the region common to $y \geq x$ and $x + y < 2$.

Solution

(a) $y \geq x$

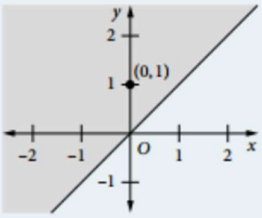
Test the point $(0, 1)$

Test: LHS ≥ RHS using (0,1)

$y > -x$, sub in $y = 1$ and $x = 0$

$1 > = 0$, which is true.

Shade region above line



(b) $x + y < 2$

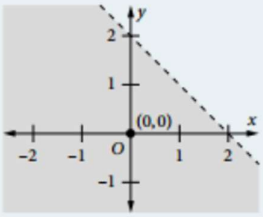
Test the point $(0, 0)$

Test LHS < RHS using the point (0,0)

$x + y < 2$, where $x = 0$ and $y = 0$

$0 + 0 < 2$, which is true.

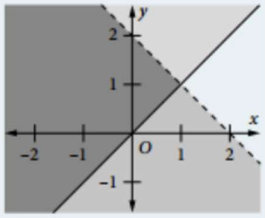
Shade region below dashed line



(c) $y \geq x$ and $x + y < 2$

Identify common region

Shade clearly, using a darker shading for the common region



Example 10

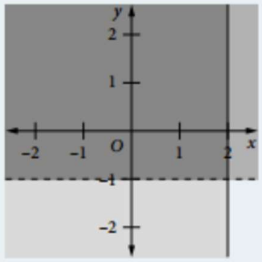
Sketch the region defined by each pair of inequalities. Describe the region in words.

(a) $x \leq 2, y > -1$ **(b)** $y \leq x + 1, x \leq 1$

Solution

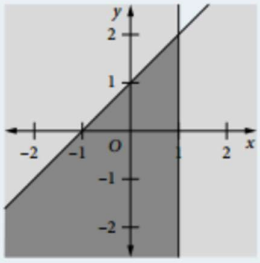
Draw each boundary and shade the two regions, then shade the common region differently.

(a) $x \leq 2, y > -1$



The region on and to the left of the line $x = 2$ that is also above the line $y = -1$.

(b) $y \leq x + 1, x \leq 1$



The region on and below the line $y = x + 1$ that is also on and to the left of the line $x = 1$.

If the shading of different regions becomes difficult to show, you should just lightly shade the original regions before darkening the final answer, as in part **(b)**.

SIMULTANEOUS LINEAR INEQUALITIES

Example 11

Sketch the region defined by the three inequalities $x - y \geq -1$, $x + 3y \geq -1$, $5x + 3y \leq 19$. Show the points of intersection of the lines. Describe the region in words.

Solution

To find the points of intersection of the lines, solve pairs of equations simultaneously:

- The lines $x - y = -1$ and $5x + 3y = 19$ intersect at $A(2, 3)$
- The lines $x - y = -1$ and $x + 3y = -1$ intersect at $B(-1, 0)$
- The lines $5x + 3y = 19$ and $x + 3y = -1$ intersect at $C(5, -2)$

The shaded region is the interior of the triangle bounded by the lines $x - y = -1$, $x + 3y = -1$ and $5x + 3y = 19$. The vertices of the region are the intersection points A, B and C.