TRIGONOMETRY - CHAPTER REVIEW

1 Simplify:

(a) $\cos(90° - \theta)$ **(b)** $\sin(270° - \theta)$ **(c)** $\cos(90° + \theta)$

$= \sin\theta$ $-\cos\theta$ $-\sin\theta$

(d) $\tan(\theta - 180°)$ **(e)** $\tan(180° - \theta)$ **(f)** $\sin(\theta + 180°)$

$\tan\theta$ $\tan(-\theta) = -\tan\theta$ $-\sin\theta$

2 Write the exact value.

(a) $\tan 315°$ **(b)** $\sin 225°$ **(c)** $\cos 180°$

$= \tan 135 = -1$ $-\sin 45 = -\sqrt{2}/2$ -1

(d) $\tan 360°$ **(e)** $\sin 60°$ **(f)** $\cos 210°$

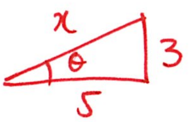
$\tan 0 = 0$ $\sqrt{3}/2$ $-\cos 30 = -\sqrt{3}/2$

3 Simplify:

(a) $\dfrac{\cos\theta}{\sin(90° - \theta)}$ **(b)** $\cos(90° + \theta) + \sin\theta$

$= \dfrac{\cos\theta}{\cos\theta} = 1$ $= -\sin\theta + \sin\theta = 0$

4 If $\tan\theta = \frac{3}{5}$ and $180° < \theta < 270°$, write the exact value of: **(a)** $\sin\theta$ **(b)** $\cos\theta$



$x^2 = 3^2 + 5^2 = 9 + 25 = 34$ so $x = \sqrt{34}$

$\sin\theta = \dfrac{3}{\sqrt{34}}$ $\cos\theta = \dfrac{5}{\sqrt{34}}$

5 If $\sin\alpha = 0.6$ and $0° < \alpha < 90°$, write the exact value of:

(a) $\sin(180° - \alpha)$ **(b)** $\cos(90° - \alpha)$ **(c)** $\cos(180° + \alpha)$

$= \sin\alpha = 0.6$ $= \sin\alpha = 0.6$

$= -\cos\alpha$

$= -\sqrt{1 - \sin^2\alpha}$

$= -\sqrt{1 - 0.6^2}$

$= -\sqrt{0.64}$

$= -0.8$

(d) $\tan\alpha$

$= \dfrac{\sin\alpha}{\cos\alpha}$

$= \dfrac{0.6}{(-0.8)} = \dfrac{-3}{4}$

(e) $\tan(180° - \alpha)$

$= \tan(-\alpha) = -\tan\alpha$

$= \dfrac{3}{4}$

(f) $\sin(360° - \alpha)$

$= \sin(-\alpha)$

$= -\sin\alpha$

$= -0.6$

6 If $\tan\theta = t$, express in terms of t:

(a) $\tan(90° - \theta)$

$= \dfrac{\sin(90-\theta)}{\cos(90-\theta)} = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\tan\theta} = \dfrac{1}{t}$

(b) $\tan(180° + \theta)$

$= \tan\theta = t$

(c) $\cot(180° - \theta)$

$= \dfrac{1}{\tan(180-\theta)} = \dfrac{1}{-\tan\theta}$

$= -\dfrac{1}{t}$

(d) $\tan(360° - \theta)$

$= \tan(-\theta) = -\tan\theta$

$= -t$

(e) $\tan(-\theta)$

$= -\tan\theta$

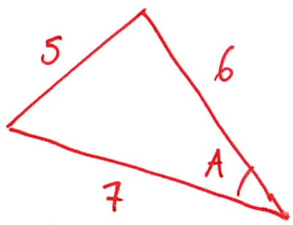
$= -t$

(f) $\tan(90° + \theta)$

$= \dfrac{\sin(90+\theta)}{\cos(90+\theta)}$

$= \dfrac{+\cos\theta}{-\sin\theta} = -\dfrac{1}{t}$

7 Calculate the cosine of the smallest angle of the triangle with side lengths 5 cm, 6 cm and 7 cm.



$5^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \cos A$

$\therefore \cos A = \dfrac{5^2 - 6^2 - 7^2}{(-2) \times 6 \times 7}$

$\therefore \cos A = \dfrac{60}{84} = \dfrac{5}{7}$

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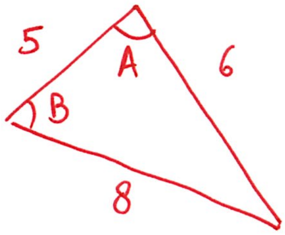
8 Find the size of the largest angle of the triangle with side lengths 5 cm, 6 cm and 8 cm. Hence, show that the triangle is obtuse-angled.



$$8^2 = 6^2 + 5^2 - 2 \times 5 \times 6 \cos A$$

$$\therefore \cos A = \frac{8^2 - 6^2 - 5^2}{(-2) \times 5 \times 6}$$

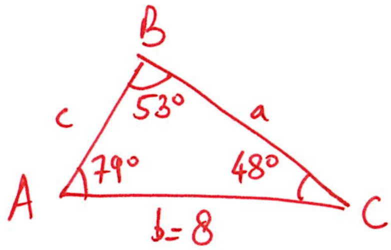
$$\cos A = \frac{-3}{60} = \frac{-1}{20}$$

as $\cos A < 0$

then $A > 90°$

9 In ΔABC, $B = 53°$, $C = 48°$, $AC = 8$ cm. Calculate:

(a) the length of BC (b) the area of ΔABC.



a) $\hat{A} = 180 - 53 - 48 = 79°$

$$\frac{BC}{\sin 79} = \frac{8}{\sin 53} \Rightarrow BC = \frac{8 \times \sin 79}{\sin 53}$$

$BC \simeq 9.8$ cm

b) $\text{Area} = \frac{1}{2} \times 8 \times 9.8 \times \sin 48 \simeq 29.1 \text{ cm}^2$

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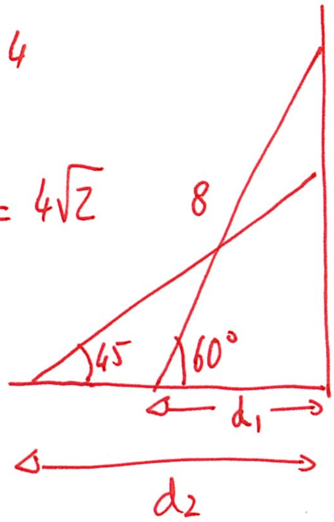
10 A ladder 8 m long rests against a wall and its foot makes an angle of 60° with the horizontal ground. The top of the ladder then slips down the wall until its foot makes an angle of 45° with the ground. Find, in simplest surd form, how far down the wall the ladder slips.

$\cos 60 = \frac{d_1}{8}$ so $d_1 = 8\cos 60 = \frac{8}{2} = 4$

$\sin 45 = \frac{d_2}{8}$ so $d_2 = 8\sin 45 = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$



So $d_2 - d_1 = 4\sqrt{2} - 4$

$= 4(\sqrt{2} - 1)$

$\simeq 1.65$ m approx

11 From a point A, level with the foot of a vertical pole and 30 m from it, the angle of elevation of the top of the pole is 40°. Calculate:

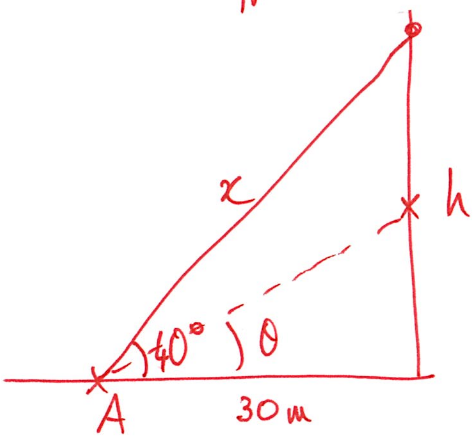
(a) the height of the pole (b) the direct distance from A to the top of the pole

(c) the angle of elevation from A of a point half-way up the pole.

a) $\tan 40 = \frac{h}{30}$ so $h = 30\tan 40 \simeq 25$ m approx

b) $\cos 40 = \frac{30}{x}$ so

$x = \frac{30}{\cos 40} = 39.1$ m approx



c) $\frac{h}{2} = \frac{30\tan 40}{2} = 15\tan 40$

so $\tan\theta = \frac{h/2}{30} = \frac{15\tan 40}{30} = \frac{\tan 40}{2}$ $\therefore \theta \simeq 22.7°$

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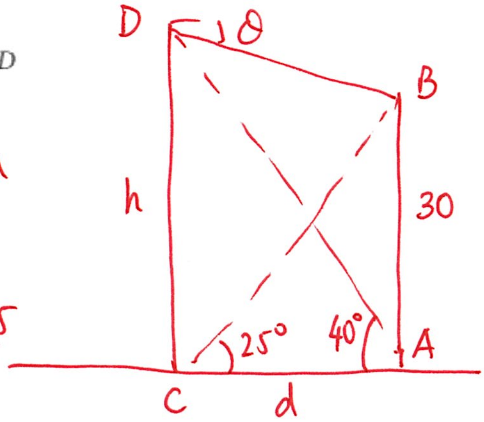
12 AB and CD are two vertical buildings with their bases at A and at C on horizontal ground. The height of AB is 30 m. The angle of elevation of B as seen from C is 25° and the angle of elevation of D as seen from A is 40°. Calculate:

(a) the horizontal distance between the buildings (b) the height of CD

(c) the angle of depression of B as seen from D.



a) $\tan 25 = \frac{30}{d}$ so $d = \frac{30}{\tan 25} \simeq 64.3$ m

b) $\tan 40 = \frac{DC}{d} = \frac{DC}{\frac{30}{\tan 25}} = \frac{DC \tan 25}{30}$

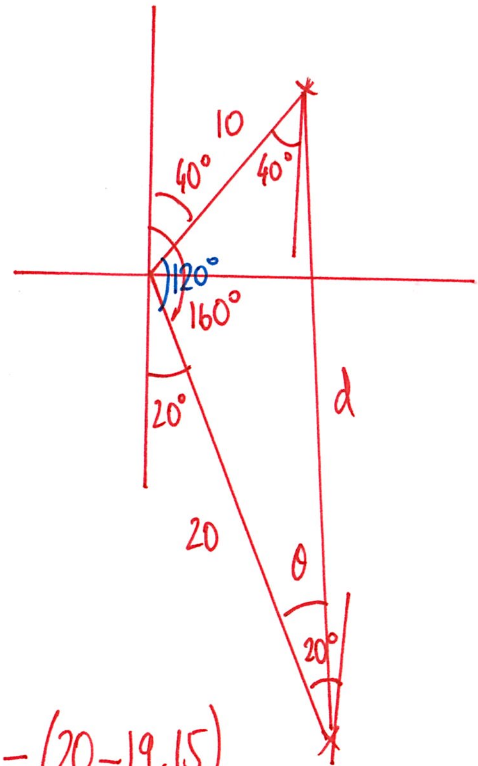
$\therefore DC = \frac{30 \tan 40}{\tan 25} \simeq 54.0$ m

c) $\tan\theta = \frac{54-30}{64.3} = 20.5°$ approx

13 Two yachts sail in a straight line from a buoy B. One sails 10 km in the direction 040° and the other sails 20 km in the direction 160°.

(a) How far apart are the yachts?

(b) What is the bearing of the first yacht as seen from the second yacht?



a) $d^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \cos 120$

$d^2 = 700 \quad \therefore d \simeq 26.4$ km

b) $\frac{\sin\theta}{10} = \frac{\sin 120}{26.4}$

$\therefore \theta = 19.15°$

Hence the bearing of the 1st yacht as seen from the second yacht is $360 - (20 - 19.15)$

$= 359°9'$

14 (a) Find a simplified expression for r given that $r^2 = (100 - 50t)^2 + (80t)^2 - 4(100 - 50t) \times 80t \times \cos 60°$.

(b) Find the value of r to the nearest whole number when $t = \frac{30}{43}$.

a) $r^2 = 10{,}000 - 10{,}000t + 2500t^2 + 6400t^2 - (400 - 200t) \times 80t \times \frac{1}{2}$

$r^2 = 10{,}000 + 8{,}900t^2 - 10{,}000t - 16{,}000t + 8000t^2$

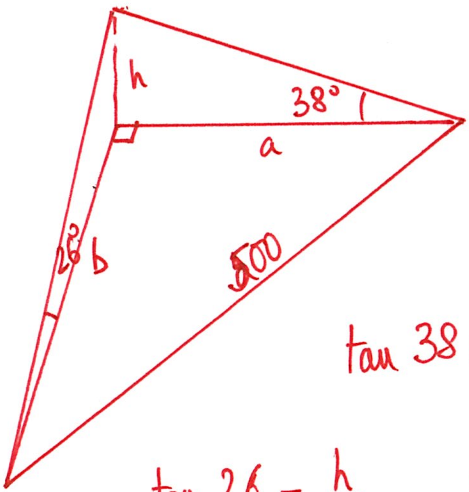
$r^2 = 16{,}900t^2 - 26{,}000t + 10{,}000 = 1{,}000\left[16.9t^2 - 26t + 10\right]$

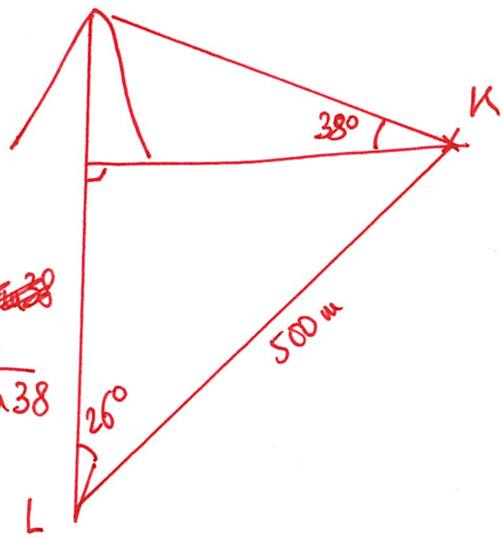
$r = 10\sqrt{10}\left[16.9t^2 - 26t + 10\right]^{1/2}$

b) $r = 10\sqrt{10}\left[16.9 \times \left(\frac{30}{43}\right)^2 - 26 \times \left(\frac{30}{43}\right) + 10\right]^{1/2}$

$r = 9.3$ so 9

15 The elevation of a hill at a place K due east of the hill is 38°; at a place L, due south of K, the elevation of the hill is 26°. If the distance from K to L is 500 metres, calculate the height of the hill to the nearest metre.





$\tan 38 = \frac{h}{a}$ ∴ $a = \frac{h}{\tan 38}$

$\tan 26 = \frac{h}{b}$ $\quad b = \frac{h}{\tan 26}$

Further $a^2 + b^2 = 500^2$

so $\frac{h^2}{\tan^2 38} + \frac{h^2}{\tan^2 26} = 500^2$

$h^2 = \frac{500^2(\tan^2 38 \tan^2 26)}{\tan^2 38 + \tan^2 26}$

$h \simeq 207$ m