INTEGRATING THE EXPONENTIAL FUNCTION

Indefinite integral of e^x

Because the exponential function e^x is its own derivative, it is also its own primitive:

$$\int e^x \, dx = e^x + C$$

Indefinite integral of e^{ax+b} (a and b constant)

$$\frac{d}{dx}(e^{ax+b}) = a\, e^{ax} \qquad \text{therefore:} \qquad \int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + C$$

Examples: $\int e^{2x+1} \, dx = \frac{1}{2} e^{2x+1} + C$

$$\int e^{\left(\frac{2x}{5}+3\right)} \, dx = \frac{5}{2} e^{\left(\frac{2x}{5}+3\right)} + C$$

Indefinite integral of $f'(x)\, e^{f(x)}$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\, e^{f(x)} \qquad \text{therefore:} \qquad \int f'(x)\, e^{f(x)} \, dx = e^{f(x)} + C$$

Indefinite integral of a^x

$$\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x \ln a}) = \ln a\, e^{x \ln a} = \ln a\, a^x \qquad \text{so:} \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C$$

Example 9

Find: **(a)** $\int e^{4x-1} \, dx$ **(b)** $\int 3x^2 e^{x^3+1} \, dx$

Solution

(a) $\frac{d}{dx}(4x-1) = 4$: $\int e^{4x-1} \, dx = \frac{1}{4} e^{4x-1} + C$

(b) $\frac{d}{dx}\left(x^3+1\right) = 3x^2$:

The integral must be of the form $\int f'(x) e^{f(x)} \, dx = e^{f(x)} + C$ where $f(x) = x^3 + 1$.

$\int 3x^2 e^{x^3+1} \, dx = e^{x^3+1} + C$