INTERSECTION OF TWO LINES

1 The coordinates of the intersection point of the lines $x + 2y - 3 = 0$ (①) and $2x - 2y - 6 = 0$ (②) are:

A $(0,3)$ B $(0,-3)$ C $(3,0)$ D $(-3,0)$

① + ② $\Rightarrow$ $3x - 9 = 0$ $\quad x = 3$

$3 + 2y - 3 = 0$ $\quad$ so $y = 0$ $\quad$ Point is $(3,0)$

2 Find the equation of the line that contains the intersection point of the lines $2x + 5y - 19 = 0$ (①) and $3x - 4y + 6 = 0$ (②) and is parallel to the line with equation $4x - y - 8 = 0$.

$3 \times ① - 2 \times ② \Rightarrow 15y - 57 + 8y - 12 = 0 \Rightarrow 23y = 69 \quad y = 3$

and $\therefore$ $2x + 5 \times 3 - 19 = 0 \Rightarrow 2x = 4 \quad x = 2$ so point $(2,3)$

$4x - y - 8 = 0 \Rightarrow y = 4x - 8 \quad m = 4$

$y - 3 = 4(x - 2) \Rightarrow \boxed{y = 4x - 5}$

5 Find the equation of the straight line that contains the intersection point of the lines $3x + 2y - 12 = 0$ (①) and $5x - y - 7 = 0$ (②) and that:

(a) passes through the point $(-4,-5)$ (b) is parallel to the line $2x - y + 4 = 0$

(c) is perpendicular to the line $y = 5$.

① + 2 × ② $\Rightarrow 13x - 12 - 14 = 0 \Rightarrow x = 2$

and from there: $3 \times 2 + 2y - 12 = 0 \Rightarrow y = \frac{1}{2} \times (+6) = 3$

so the point of intersection is $(2,3)$

a) $m = \frac{-5-3}{-4-2} = \frac{-8}{-6} = \frac{4}{3}$ $\quad y - 3 = \frac{4}{3}(x - 2)$ $\quad y = 4/3\,x + 1/3$

b) $y = 2x + 4 \quad m = 2$ $\quad y - 3 = 2(x - 2)$

$\Leftrightarrow y = 2x - 1$

c) $x = 2$

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$D(x_D, y_D)$

7 $ABCD$ is a quadrilateral. The coordinates of A, B and C are $(-8,6)$, $(2,4)$ and $(5,-7)$ respectively. If the diagonals are perpendicular and DC is parallel to the x-axis, find:

(a) the coordinates of D **(b)** the coordinates of the intersection point of the diagonals.

$m_{AC} = \frac{-7-6}{5+8} = \frac{-13}{13} = -1$ $\quad m_{BD} = \frac{y_D - 4}{x_D - 2} = 1$

so $y_D - 4 = x_D - 2$ so $y_D = x_D + 2$

Further DC // to x-axis so $m_{DC} = 0 = \frac{y_D + 7}{\quad}$ so $y_D = -7$

so $x_D = -7 - 2 = -9$ $\quad D(-9, -7)$

b) equation of (AC) is $y - 6 = -(x+8) \iff y = -x - 2$

equation of (BD) is $y - 4 = x - 2 \iff y = x + 2$

The lines intersect at $2y = 0$ i.e. $y = 0$

and $x = -2$ so at $(-2, 0)$

9 Without actually solving the simultaneous equations, state whether the following pairs of lines intersect, are parallel or coincide.

(a) $2x - 3y - 8 = 0$ ① $\quad 4x - 6y - 16 = 0$ ②

(b) $x + 3y + 7 = 0 \quad 2x + 7y + 16 = 0$

(c) $6x - 5y - 24 = 0 \quad 9x - 4y - 22 = 0$

(d) $x + y - 7 = 0 \quad x + y - 8 = 0$

a) Eq ② is the same as ①, divided by 2. So coincide

b) $m_① = -1/3 \quad m_② = -2/7$ so intersect

c) $m_① = 6/5 \quad m_② = 9/4$ so intersect

d) $m_① = -1 \quad m_② = -1$ so parallel.