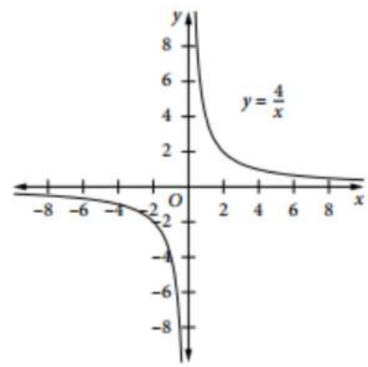
THE EQUATION $y = k / x$ AND INVERSE VARIATION

Earlier you have seen the link between direct variation and the equation of the straight line $y = kx$, which passes through the origin. By definition, two variables are in direct variation if one is a constant multiple of the other. This means if one variable increases then the other variable also increases at the same rate. This can also be written as $\frac{y}{x} = k$: the ratio between the two variables is a constant.

Similarly, the equation $xy = k$ can be written as $y = \frac{k}{x}$. In this situation, as x increases then y decreases; or, as x decreases then y increases. A change in one variable produces the opposite change in the other variable. This is called **inverse variation** or inverse proportion. This is expressed by saying that 'y is inversely proportional to x, where k is the constant of proportion (or variation)'.

Inverse variation has many applications in science. For example, in physics, Boyle's law states that 'at constant temperature a fixed mass of gas occupies a volume inversely proportional to the pressure exerted on it'. This is written as the formula $V = \frac{k}{P}$ or $PV = k$, where k is a constant.

The equation $y = \frac{k}{x}$



The equation $y = \frac{k}{x}$ is the same as $xy = k$, the rectangular hyperbola.
Consider the graph of $y = \frac{4}{x}$:

- the domain of the function is real x, $x \neq 0$
- the range is real y, $y \neq 0$
- the line $x = 0$ is a vertical asymptote
- the line $y = 0$ is a horizontal asymptote.

An **asymptote** is a line that the curve approaches but never meets.
In both branches of the curve, as x increases then y decreases.
Notice that this graph passes through the points (2, 2) and (−2, −2).
In general, the graph of $y = \frac{k}{x}$ will pass through the points $\left(\sqrt{k}, \sqrt{k}\right)$ and $\left(-\sqrt{k}, -\sqrt{k}\right)$.

Example 24

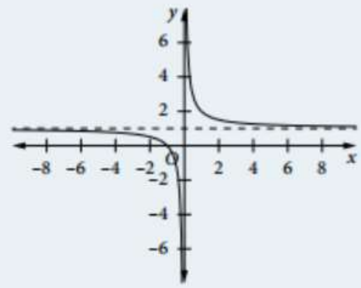
Sketch the following graphs. Give the domain and range of each function and state the equations of any asymptotes.

(a) $y = \frac{x+1}{x}$ **(b)** $y = \frac{x}{x+1}$

Solution

(a) $y = \frac{x+1}{x}$: This may be rewritten as $y = 1 + \frac{1}{x}$.

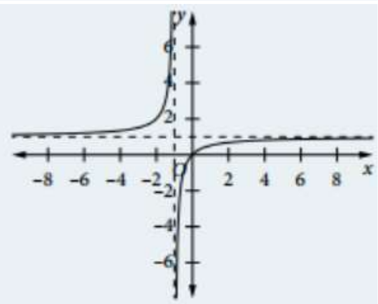
Thus the graph of $y = \frac{x+1}{x}$ is just the graph of $y = \frac{1}{x}$ moved up 1 unit.



Vertical asymptote is $x = 0$.
Horizontal asymptote is $y = 1$.
Domain is real x, $x \neq 0$.
Range is real y, $y \neq 1$.

(b) $y = \frac{x}{x+1}$: This may be rewritten as $y = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$.

Vertical asymptote is $x = -1$.
Horizontal asymptote is $y = 1$.
Domain is real x, $x \neq -1$.
Range is real y, $y \neq 1$.



Note: The equations in this example do not represent inverse variation as their graphs do not have the coordinate axes as their asymptotes.