EXPONENTIALS AND LOGARITHMS - CHAPTER REVIEW II

1. Find the value of y.

   (1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \dfrac{1}{8} = y$

   (5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \dfrac{1}{7} = y$ (8) $\log_3 \dfrac{1}{9} = y$

   (9) $\log_y 32 = 5$ (10) $\log_9 y = -\dfrac{1}{2}$ (11) $\log_4 \dfrac{1}{8} = y$ (12) $\log_9 \dfrac{1}{81} = y$

2. Evaluate.

   (1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (3) $\log_{25} 5^3$ (4) $16^{\log_4 8}$

3. Write the following expressions in terms of logs of x, y and z.

   (1) $\log x^2 y$ (2) $\log \dfrac{x^3 y^2}{z}$ (3) $\log \dfrac{\sqrt{x}\,\sqrt[3]{y^2}}{z^4}$ (4) $\log xyz$

   (5) $\log \dfrac{x}{yz}$ (6) $\log \left(\dfrac{x}{y}\right)^2$ (7) $\log (xy)^{\frac{1}{3}}$ (8) $\log x\sqrt{z}$

(9) $\quad \log \dfrac{\sqrt[3]{x}}{\sqrt[3]{yz}}$ $\qquad$ (10) $\quad \log \sqrt[4]{\dfrac{x^3 y^2}{z^4}}$ $\qquad$ (11) $\quad \log x \sqrt{\dfrac{\sqrt{x}}{z}}$ $\qquad$ (12) $\quad \log \sqrt{\dfrac{xy^2}{z^8}}$

4. Write the following equalities in exponential form.

(1) $\quad \log_3 81 = 4$ $\qquad$ (2) $\quad \log_7 7 = 1$ $\qquad$ (3) $\quad \log_{\frac{1}{2}} \dfrac{1}{8} = 3$ $\qquad$ (4) $\quad \log_3 1 = 0$

(5) $\quad \log_4 \dfrac{1}{64} = -3$ $\qquad$ (6) $\quad \log_6 \dfrac{1}{36} = -2$ $\qquad$ (7) $\quad \log_x y = z$ $\qquad$ (8) $\quad \log_m n = \dfrac{1}{2}$

5. Write the following equalities in logarithmic form.

(1) $\quad 8^2 = 64$ $\qquad$ (2) $\quad 10^3 = 1000$ $\qquad$ (3) $\quad 4^{-2} = \dfrac{1}{16}$ $\qquad$ (4) $\quad 3^{-4} = \dfrac{1}{81}$

(5) $\quad \left(\dfrac{1}{2}\right)^{-5} = 32$ $\qquad$ (6) $\quad \left(\dfrac{1}{3}\right)^{-3} = 27$ $\qquad$ (7) $\quad x^{2z} = y$ $\qquad$ (8) $\quad \sqrt{x} = y$

6. True or False?

(1) $\log\left(\dfrac{x}{y^3}\right) = \log x - 3\log y$ (2) $\log(a-b) = \log a - \log b$ (3) $\log x^k = k \cdot \log x$

(4) $(\log a)(\log b) = \log(a+b)$ (5) $\dfrac{\log a}{\log b} = \log(a-b)$ (6) $(\ln a)^k = k \cdot \ln a$

(7) $\log_a a^a = a$

(8) $-\ln\left(\dfrac{1}{x}\right) = \ln x$

7. Solve the following logarithmic equations.

(1) $\ln x = -3$

(2) $\log_2(3x - 2) = 2$

(3) $2\log x = \log 2 + \log(3x - 4)$

(4) $\log x + \log(x - 1) = \log(4x)$

(5) $\log_3(x + 25) - \log_3(x - 1) = 3$ (6) $\log_9(x - 5) + \log_9(x + 3) = 1$

(7) $\log_{10} x + \log_{10}(x - 3) = 1$ (8) $\log_2(x - 2) + \log_2(x + 1) = 2$

8. Prove the following statements.

   (1) $\log_{\sqrt{b}} x = 2 \log_b x$ (2) $\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$ (3) $\log_{b^4} x^2 = \log_b \sqrt{x}$

9. Given that $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, express the following expressions in terms of x, y, and z.

(1) $\log 12$

(2) $\log 392$

(3) $\log\left(\frac{14}{3}\right)$

(4) $\log\left(\frac{6}{7}\right)$

(5) $\log 1.5$

(6) $\log 10.5$

(7) $\log 24.5$

(8) $\log\left(\frac{7776}{7}\right)$

10. Solve the following equations.

(1) $3^x - 2 = 12$

(2) $3^{1-x} = 2$

(3) $4^x = 5^{x+1}$

(4) $6^{1-x} = 10^x$

(5) $3^{2x+1} = 2^{x-2}$

(6) $\dfrac{10}{1 + e^{-x}} = 2$

13. 15 000\$ is invested in an account that yeilds 5% interest per year. After how many years will the account be worth 91 221.04\$ if the interest is compounded yearly?

14. 8 000\$ is invested in an account that yeilds 6% interest per year. After how many years will the account be worth 13709.60\$ if the interest is compounded monthly?

15. Starting at the age of 40, an average man loses 5% of his hair every year. At what age should an average man expect to have half his hair left?

16. A bacteria culture starts with 10 00 bacteria and the number doubles every 40 minutes.

    (a) Find a formula for the number of bacteria at time t.

    (b) Find the number of bacteria after one hour.

    (c) After how many minutes will there be 50 000 bacteria?