USING VECTORS IN GEOMETRIC PROOFS

3. OABC is a parallelogram in which $\overrightarrow{OA} = 6\vec{i}$ and $\overrightarrow{OC} = \vec{i} + 3\vec{j}$. Find:

a) $\overrightarrow{AB}$ and $\overrightarrow{CB}$

b) the diagonal vectors $\overrightarrow{OB}$ and $\overrightarrow{CA}$

c) the vectors $\overrightarrow{ON}$ and $\overrightarrow{OM}$, where N is the midpoint of OB and M is the midpoint of CA. What conclusion can you make?

d) the vectors $\overrightarrow{CP}$ and $\overrightarrow{BP}$, where P is the midpoint of OA

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6 *OABC* is a quadrilateral, $\overrightarrow{OA} = 4\underset{\sim}{i}$, $\overrightarrow{OB} = 6\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\overrightarrow{OC} = 8\underset{\sim}{j}$.

(a) If P and Q are the midpoints of AB and BC respectively, find $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$.

(b) Show that $\overrightarrow{PQ} = k\overrightarrow{AC}$. What geometrical conclusion can you now make?

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7 The position vectors of the vertices A, B, C and D of a quadrilateral are $\underset{\sim}{i} + 2\underset{\sim}{j}$, $5\underset{\sim}{i} + 2\underset{\sim}{j}$, $4\underset{\sim}{i} - \underset{\sim}{j}$ and $2\underset{\sim}{i} - \underset{\sim}{j}$ respectively. Show that the diagonals intersect at right angles.

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15 P and Q are the midpoints of the sides OA and OB of triangle OAB. Use a vector method to prove that PQ is parallel to AB and half its length.

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17 O is any point on the diagonal BD of the parallelogram $ABCD$. Prove that $\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OD}$.

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21 $ABCD$ is a trapezium in which $AB = x$, $DC = 2x$, $DA = y$. If E is a point in BC such that $BE = \frac{1}{3}BC$, prove that $\overrightarrow{AC} \bullet \overrightarrow{DE} = \frac{2}{3}\left(4x^2 - y^2\right)$.

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Prove that:

26 The midpoint of the hypotenuse of a right-angled triangle is equidistant from the three vertices.

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Prove that:

27 If the diagonals of a parallelogram are equal in length, then the parallelogram is a rectangle.

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Prove that:

31 For any triangle PQR, $\left|\overrightarrow{PQ}\right| = \left|\overrightarrow{PR}\right| \cos P + \left|\overrightarrow{QR}\right| \cos Q$.

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Prove that:

32 The sum of the squares of the distances from a point A to two opposite vertices of a rectangle is equal to the sum of the squares of the distances from A to the remaining two vertices.