APPROXIMATE METHODS OF INTEGRATION - TRAPEZOIDAL RULE

2 Evaluate $\int_0^1 \frac{dx}{x^2+1}$ using the trapezoidal rule with: **(a)** two subintervals **(b)** four subintervals.

a) $\int_0^1 \frac{dx}{x^2+1} \approx \frac{1-0}{4}\left[f(0) + 2\times f\left(\frac{0+1}{2}\right) + f(1)\right]$

$\text{——} \approx \frac{1}{4}\left[1 + 2\times\frac{1}{\left(\frac{1}{2}\right)^2+1} + \frac{1}{1^2+1}\right]$

$\text{——} \approx \frac{1}{4}\left[1 + 2\times\frac{4}{5} + \frac{1}{2}\right] = \frac{1}{4}\left[\frac{10+16+5}{10}\right] = \frac{31}{40} = 0.775$

b) $\int_0^1 \frac{dx}{x^2+1} \approx \frac{1-0}{2\times 4}\left[f(0) + 2f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 2f\left(\frac{3}{4}\right) + f(1)\right]$

$f\left(\frac{1}{4}\right) \approx \frac{1}{\left(\frac{1}{4}\right)^2+1} = \frac{16}{17}$ $\quad f\left(\frac{1}{2}\right) = \frac{1}{\left(\frac{1}{2}\right)^2+1} = \frac{4}{5}$ $\quad f\left(\frac{3}{4}\right) = \frac{1}{\left(\frac{3}{4}\right)^2+1} = \frac{16}{25}$

$\int_0^1 \frac{dx}{x^2+1} \approx \frac{1}{8}\left[1 + 2\times\frac{16}{17} + 2\times\frac{4}{5} + 2\times\frac{16}{25} + \frac{1}{2}\right]$

$\int_0^1 \frac{dx}{x^2+1} \approx \frac{1}{8}\left[1 + \frac{32}{17} + \frac{8}{5} + \frac{32}{25} + \frac{1}{2}\right]$

$\int_0^1 \frac{dx}{x^2+1} \approx \frac{1}{8}\times\frac{5323}{850}$

$\int_0^1 \frac{dx}{x^2+1} \approx \frac{5323}{6800} = 0.7828$

APPROXIMATE METHODS OF INTEGRATION - TRAPEZOIDAL RULE

4 Evaluate $\int_1^2 x^3\,dx$ by: **(a)** direct integration **(b)** using the trapezoidal rule with two subintervals.

a) $\int_1^2 x^3\,dx = \left[\frac{x^4}{4}\right]_1^2 = \frac{2^4}{4} - \frac{1^4}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4} = 3.75$

b) $\int_1^2 x^3\,dx \approx \frac{2-1}{2\times 2}\left[f(1) + 2f\left(\frac{1+2}{2}\right) + f(2)\right]$

$\text{———} \approx \frac{1}{4}\left[1^3 + 2\times\left(\frac{3}{2}\right)^3 + 2^3\right]$

$\text{———} \approx \frac{1}{4}\left[1 + 2\times\frac{27}{8} + 8\right]$

$\text{———} \approx \frac{63}{16} = 3.9375$

APPROXIMATE METHODS OF INTEGRATION - TRAPEZOIDAL RULE

13 Use the trapezoidal rule with two subintervals to estimate $\int_0^\pi \sqrt{\sin x}\,dx$ correct to 2 decimal places.

$$\int_0^\pi \sqrt{\sin x}\;dx \approx \frac{\pi - 0}{2\times 2}\left[f(0) + 2f\left(\frac{0+\pi}{2}\right) + f(\pi)\right]$$

$$\approx \frac{\pi}{4}\left[\underbrace{\sqrt{\sin(0)}}_{=0} + 2\sqrt{\sin(\pi/2)} + \underbrace{\sqrt{\sin(\pi)}}_{=0}\right]$$

$$\approx \frac{\pi}{4} \times 2\sqrt{\sin(\pi/2)}$$

$$\approx \frac{\pi}{2}\sqrt{1}$$

$$\approx \frac{\pi}{2} \text{ so approx } 1.57$$

APPROXIMATE METHODS OF INTEGRATION - TRAPEZOIDAL RULE

17 Use the trapezoidal rule with four subintervals to evaluate $\int_0^{0.8} xe^{-x}\,dx$.

$$\int_0^{0.8} x e^{-x}\,dx \approx \frac{0.8-0}{2\times 4}\left[f(0)+2f\left(\frac{0+0.8}{4}\right)+2f\left(\frac{0+0.8}{2}\right)+2f\left(\frac{0+0.8}{4}\times 3\right)+f(0.8)\right]$$

$$\int_0^{0.8} x e^{-x}\,dx \approx 0.1\left[f(0)+2f(0.2)+2f(0.4)+2f(0.6)+f(0.8)\right]$$

$f(0) = 0 \times e^{-0} = 0$ $\qquad$ $f(0.8) = 0.8\,e^{-0.8}$

$f(0.2) = 0.2\,e^{-0.2}$ $\qquad$ $f(0.4) = 0.4\,e^{-0.4}$ $\qquad$ $f(0.6) = 0.6 \times e^{-0.6}$

$$\int_0^{0.8} x e^{-x}\,dx \approx 0.1\left[2\times 0.2\,e^{-0.2}+2\times 0.4\,e^{-0.4}+2\times 0.6\,e^{-0.6}+0.8\,e^{-0.8}\right]$$

$$\int_0^{0.8} x e^{-x}\,dx \approx 0.1\left[0.4\,e^{-0.2}+0.8\,e^{-0.4}+1.2\,e^{-0.6}+0.8\,e^{-0.8}\right]$$

$$\int_0^{0.8} x e^{-x}\,dx \approx 0.188$$