DIRECTION FIELDS

1 Consider the differential equation $\frac{dy}{dx} = y - \frac{x}{2}$.

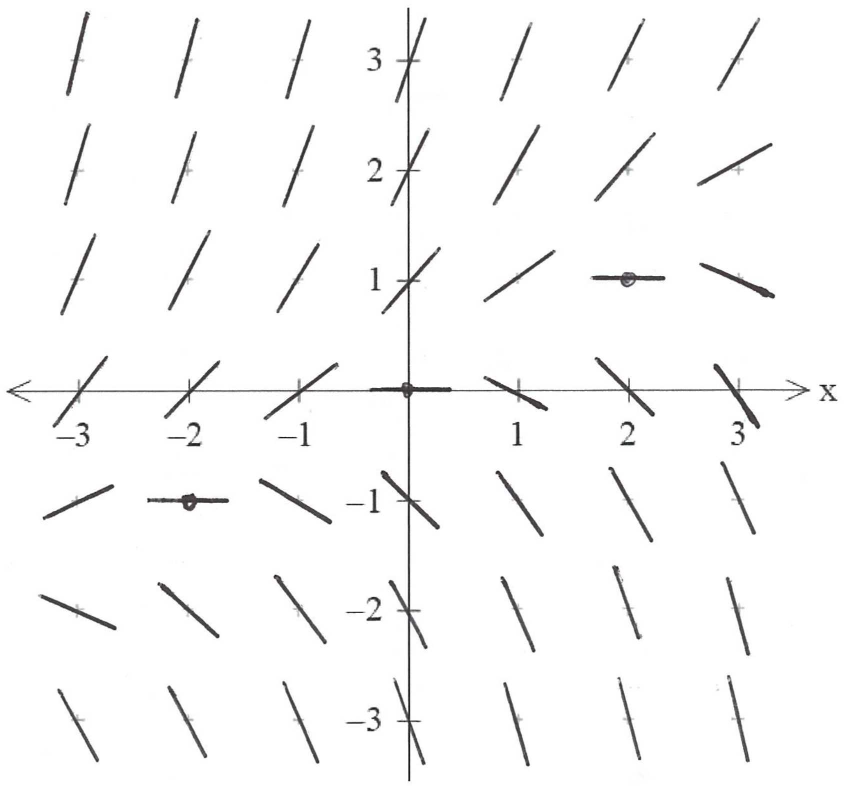
(a) Find the gradient of a solution curve at the point $(3, -2)$, assuming the curve goes through this point.

(b) Use integer values of x and y from -3 to 3 to construct a direction field for the differential equation $\frac{dy}{dx} = y - \frac{x}{2}$.

a) at $(3, -2)$ $\quad x = 3$ and $y = -2$ so $\frac{dy}{dx} = -2 - \frac{3}{2} = -\frac{7}{2} = -3.5$

b)

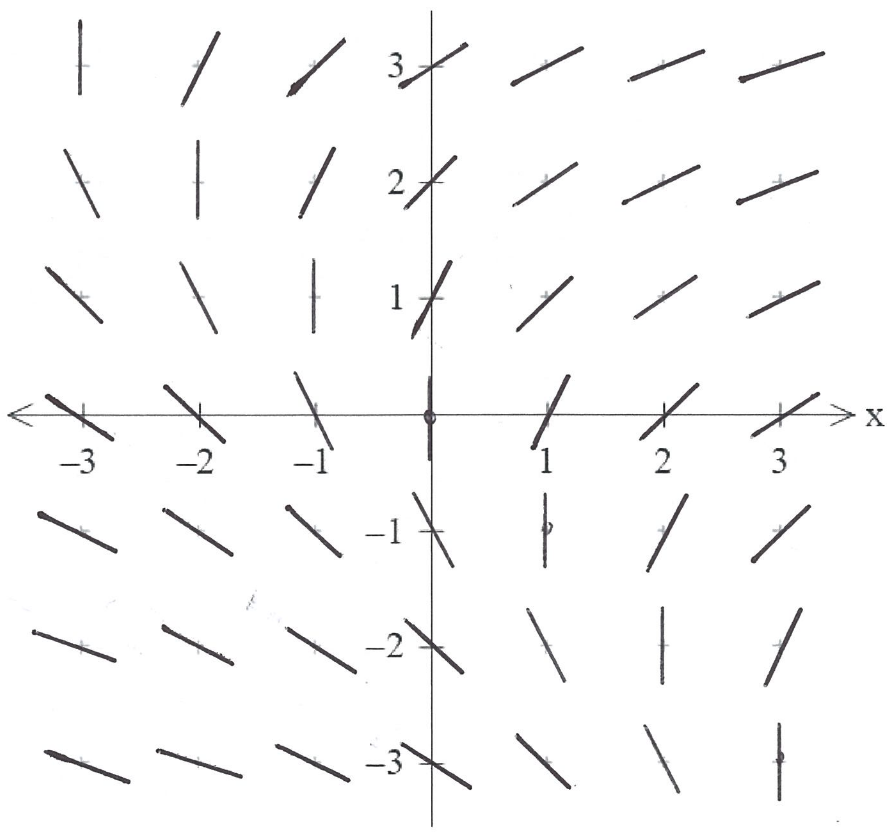
	-3	-2	-1	0	1	2	3
3	4.5	4	3.5	3	2.5	2	1.5
2	3.5	3	2.5	2	1.5	1	0.5
1	2.5	2	1.5	1	0.5	0	-0.5
0	1.5	1	0.5	0	-0.5	-1	-1.5
-1	0.5	0	-0.5	-1	-1.5	-2	-2.5
-2	-0.5	-1	-1.5	-2	-2.5	-3	-3.5
-3	-1.5	-2	-2.5	-3	-3.5	-4	-4.5



2 Construct direction fields for the following differential equations for $-3 \le x \le 3$ and $-3 \le y \le 3$. Use integer values of x and y.

(b) $\dfrac{dy}{dx} = \dfrac{2}{x+y}$

	-3	-2	-1	0	1	2	3
3	∞	2	1	2/3	1/2	2/5	1/3
2	-2	∞	2	1	2/3	1/2	2/5
1	-1	-2	∞	2	1	2/3	1/2
0	-2/3	-1	-2	∞	2	1	2/3
-1	-1/2	-2/3	-1	-2	∞	2	1
-2	-2/5	-1/2	-2/3	-1	-2	∞	2
-3	-1/3	-2/5	-1/2	-2/3	-1	-2	∞

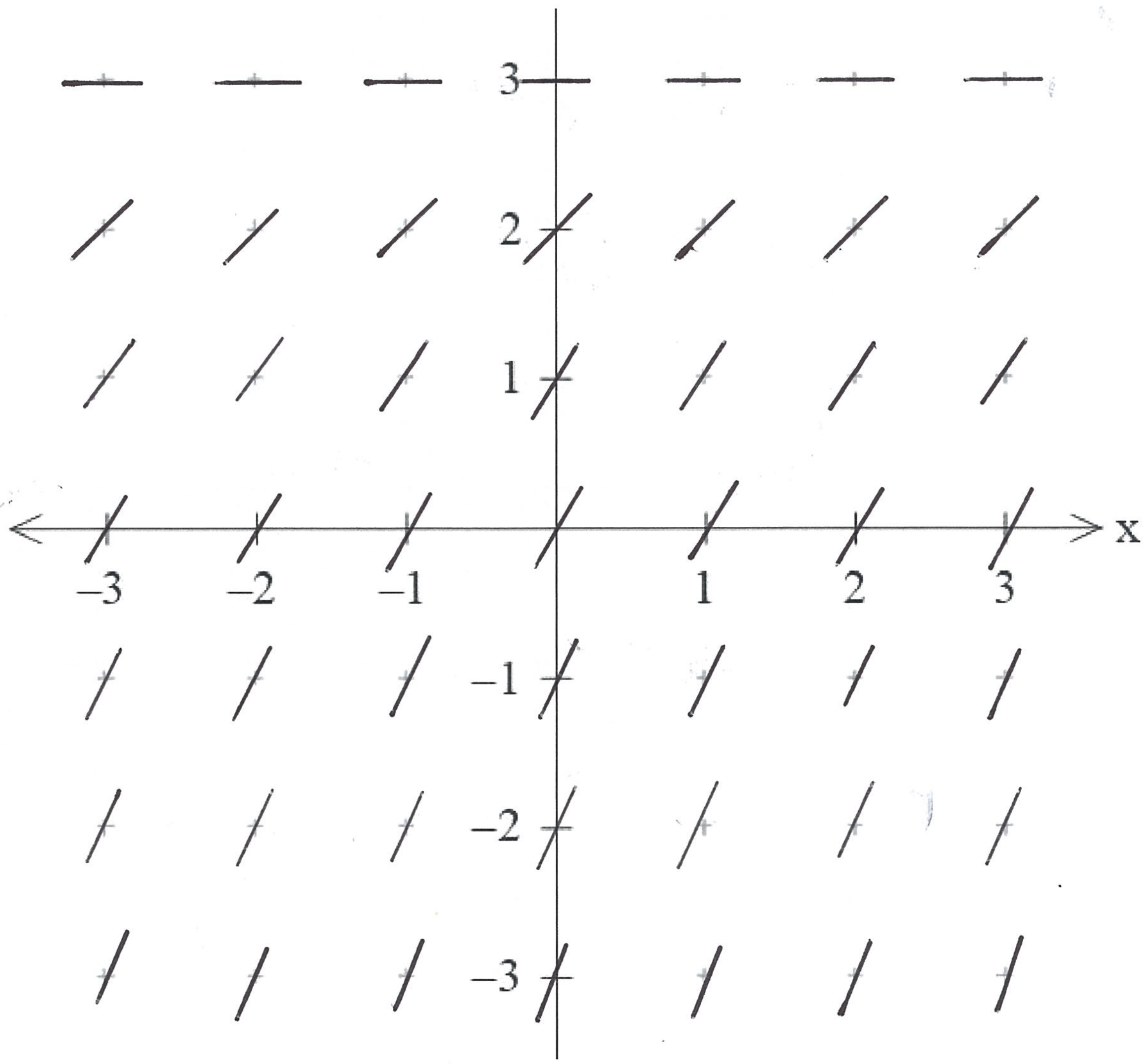


DIRECTION FIELDS

2 Construct direction fields for the following differential equations for $-3 \le x \le 3$ and $-3 \le y \le 3$. Use integer values of x and y.

(g) $\frac{dy}{dx} = \sqrt{3-y}$ only depends of y, not of x

y	-3	-2	-1	0	1	2	3
$\frac{dy}{dx}$	$\sqrt{6}$ ~ 2.4	$\sqrt{5}$ ~ 2.2	2	$\sqrt{3}$ ~ 1.7	$\sqrt{2}$ ~ 1.4	1	0



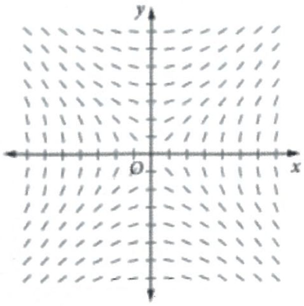
DIRECTION FIELDS

4 The graph shown is the slope field of a first-order differential equation. This differential equation could be:

A ~~$y' = \frac{y}{x}$~~ $\boxed{\text{B}}$ $y' = \frac{x}{y}$

C ~~$y' = -\frac{y}{x}$~~ D $y' = -\frac{x}{y}$

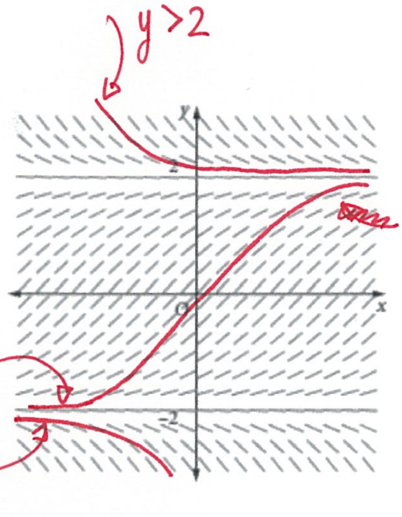
as slope around line $y=x$ is 1 not (-1)



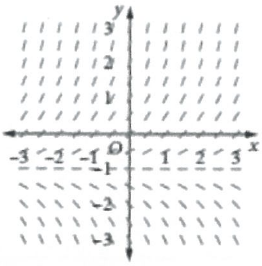
5 The slope field of $\frac{dy}{dx} = f(y)$ is shown.

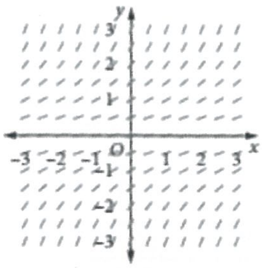
For each of the following, sketch a possible curve which is a solution to this differential equation, containing a point for which:

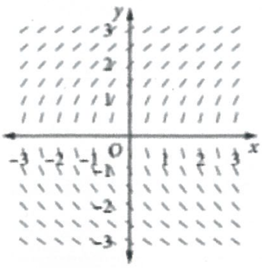
(a) $y > 2$
(b) $-2 < y < 2$
(c) $y < -2$

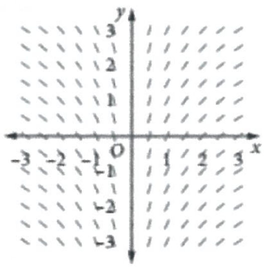


6 Which of the following slope fields does not represent a differential equation of the form $\frac{dy}{dx} = f(y)$?

A


B


C


D


$\frac{dy}{dx} = f(y)$ ∴ only varies with y

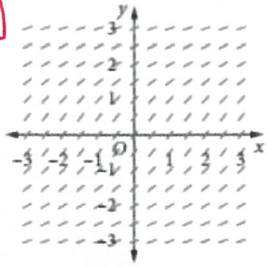
If y stays the same, then $\frac{dy}{dx}$ is constant.

That's not the case for $\boxed{\text{D}}$ as $\frac{dy}{dx}$ varies for y constant for that graph.

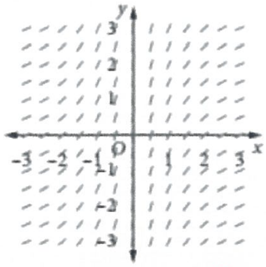
DIRECTION FIELDS

7 Which of the following slope fields represents a differential equation of the form $\frac{dy}{dx} = f(y)$?

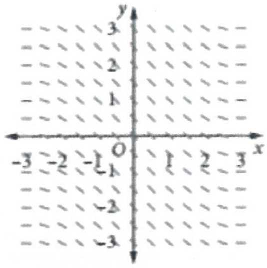
$\boxed{\text{A}}$



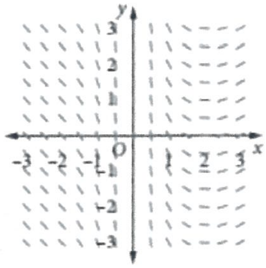
B



C



D



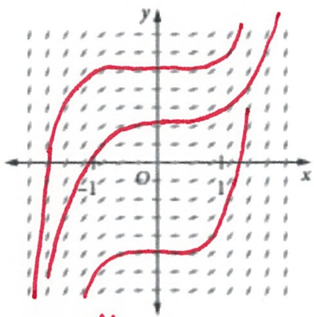
$\frac{dy}{dx} = f(y)$ $\therefore$ is constant for y constant

it can only be $\boxed{A}$

8 A first-order differential equation has a slope field as shown.

(a) Sketch three possible solutions for this differential equation.

(b) Which of the following first-order differential equations is consistent with the slope field shown?



A $\frac{dy}{dx} = xy$

$\boxed{\text{B}}$ $\frac{dy}{dx} = x^2$ slope only positive

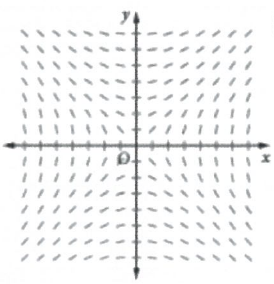
C $\frac{dy}{dx} = x^3$

D $\frac{dy}{dx} = x + y$

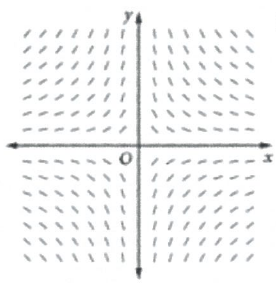
Not $\boxed{A}$ as for $y = 0$ (along x-axis), slopes are not all zero.

9 The slope field of $xy' - y = 0$ could be:

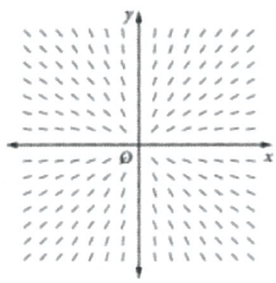
A



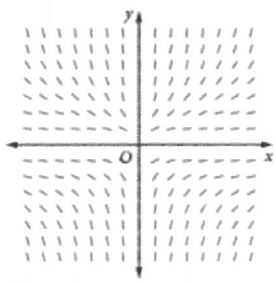
B



C



D



$y' = \frac{y}{x}$ so when $x \to 0$ $y' \to +\infty$ (vertical slopes)

when $y \to 0$ $y' = 0$ (horizontal slope), so could be B, C, D

when $y = x$ $\frac{dy}{dx} = 1$ so $\boxed{C}$

DIRECTION FIELDS

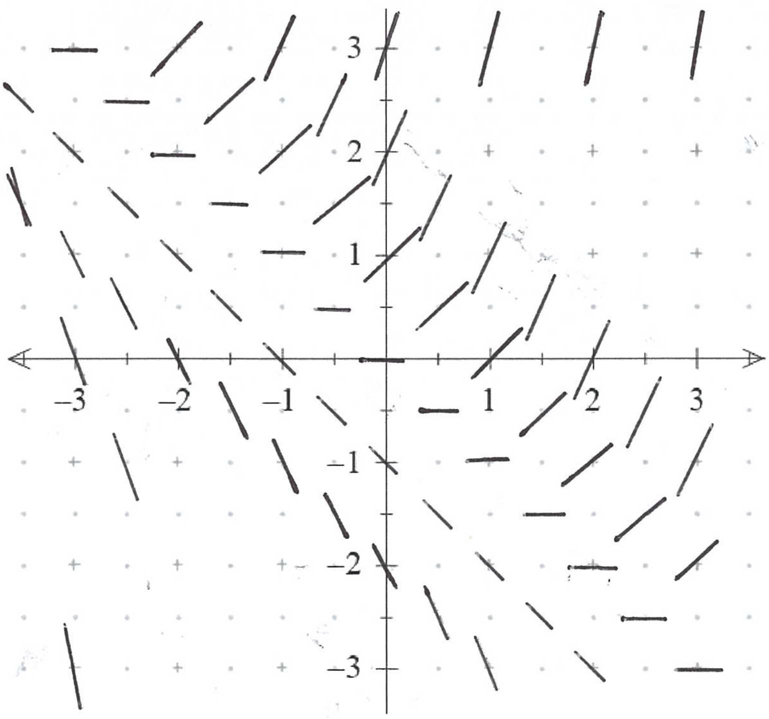
10 **(a)** Construct the direction field for the differential equation $\frac{dy}{dx} = x + y$, for $-3 \le x \le 3$ and $-3 \le y \le 3$, with x and y increasing in steps of 0.5.

(b) Draw some possible solutions to the differential equation $\frac{dy}{dx} = x + y$, including one that is a straight line, and including one that touches but does not cross the x-axis.

(c) Write the equation of the possible straight line solution.

(d) Verify whether the straight line represents a solution to the differential equation.

	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
3	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
2.5	-0.5	0		1		2		3		4		5	5.5
2	-1		0		1		2		3		4		5
1.5	-1.5	-1		0		1		2		3		4	4.5
1	-2		-1		0		1		2		3		4
0.5	-2.5	-2		-1		0		1		2		3	3.5
0	-3		-2		-1		0		1		2		3
-0.5	-3.5	-3		-2		-1		0		1		2	2.5
-1	-4		-3		-2		-1		0		1		2
-1.5	-4.5	-4		-3		-2		-1		0		1	1.5
-2	-5		-4		-3		-2		-1		0		1
-2.5	-5.5	-5		-4		-3		-2		-1		0	0.5
-3	-6	-5.5	-5	-4.5	-4	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0



c) it looks like $y = -x - 1$ is a possible solution

d) $\frac{dy}{dx} = -1$ with $y = -x - 1$

whereas $x + y = -1$

So indeed $y = -x - 1$ is a solution to

$$\frac{dy}{dx} = x + y$$