THE INDEFINITE INTEGRAL

1 Find: **(a)** $\int x\,dx$ **(b)** $\int (x^2+x+1)\,dx$ **(c)** $\int (3-x^2)\,dx$

a) $\int x\,dx = \frac{x^2}{2} + C$

b) $\int (x^2+x+1)\,dx = \frac{x^3}{3} + \frac{x^2}{2} + x + C$

c) $\int (3-x^2)\,dx = 3x - \frac{x^3}{3} + C$

(d) $\int (6x^5 - 4x^3 + 2x)\,dx$ **(e)** $\int dx$ **(f)** $\int x^n\,dx$

d) $\int (6x^5 - 4x^3 + 2x)\,dx = 6\frac{x^6}{6} - 4\frac{x^4}{4} + x^2 + C = x^6 - x^4 + x^2 + C$

e) $\int dx = x + C$

f) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

2 Find: **(a)** $\int \sqrt{x}\,dx$ **(b)** $\int \frac{1}{x^2}\,dx$ **(c)** $\int \left(1+\sqrt{x}+x\right)dx$

a) $\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

b) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{(-2+1)} + C = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

c) $\int \left(1+\sqrt{x}+x\right)dx = \int \left(1 + x^{1/2} + x\right)dx$

$= x + \frac{x^{3/2}}{3/2} + \frac{x^2}{2} + C$

$= x + \frac{2}{3}x^{3/2} + \frac{x^2}{2} + C$

3 $\int \left(1+2x+3x^2\right)dx$ is equal to:

A $x+x^2+\frac{x^3}{3}+C$ **B** $x+\frac{x^2}{2}+\frac{x^3}{3}+C$ **C** $x+x^2+x^3+C$ **D** $2+6x+C$

$\int 1 + 2x + 3x^2 = x + x^2 + x^3 + C$ so $\boxed{C}$

4 If $\frac{dy}{dx} = 1 + x + 3x^2$, find the equation of the curve that passes through the point $(2,6)$.

$$y = \int (1 + x + 3x^2)\, dx = x + \frac{x^2}{2} + x^3 + C$$

At $x = 2$, $y = 6 = 2 + \frac{2^2}{2} + 2^3 + C$

So $6 = 2 + 2 + 8 + C = 12 + C$

so $C = -6$

$$y = x + \frac{x^2}{2} + x^3 - 6$$

5 If $\frac{dy}{dx} = 1 + \sqrt{x}$, find the equation of the curve that passes through the point $(4, 10)$.

$$y = \int (1 + \sqrt{x})\, dx = \int (1 + x^{1/2})\, dx = x + \frac{x^{3/2}}{3/2} + C$$

So $y = x + \frac{2}{3} x^{3/2} + C = x + \frac{2}{3} x\sqrt{x} + C$

At $x = 4$, $y = 10$ so $10 = 4 + \frac{2}{3} \times 4 \times \sqrt{4} + C$

So $10 = 4 + \frac{16}{3} + C$ $\qquad C = 6 - \frac{16}{3}$

So $C = 2/3$

$$y = x + \frac{2}{3} x\sqrt{x} + \frac{2}{3}$$

sin cos
cos −sin

PRIMITIVES OF TRIGONOMETRIC FUNCTIONS

1 Write the primitive function of:

(a) $\sin 2x$ (b) $\cos 3x$ (c) $\sec^2 x$ (d) $\sin x + \cos x$

(e) $2\sin x - 3\cos x$ (f) $\sin\left(x+\frac{\pi}{4}\right)$ (g) $\cos\frac{x}{2}$ (h) $2\sin 2x$

a) $\int \sin 2x \, dx = -\frac{\cos 2x}{2} + C$

b) $\int \cos 3x \, dx = \frac{\sin 3x}{3} + C$

c) $\int \sec^2 x \, dx = \tan x + C$

d) $\int (\sin x + \cos x) dx = -\cos x + \sin x + C$

e) $\int (2\sin x - 3\cos x) dx = -2\cos x - 3\sin x + C$

f) $\int \sin\left(x+\frac{\pi}{4}\right) dx = -\cos\left(x+\frac{\pi}{4}\right) + C$

g) $\int \cos\left(\frac{x}{2}\right) dx = 2\sin\left(\frac{x}{2}\right) + C$

h) $\int 2\sin(2x)\, dx = 2\int \sin(2x)\, dx$

$= 2\left[-\frac{\cos(2x)}{2}\right] + C$

$= -\cos 2x + C$

2 The primitive of $3\cos\frac{x}{3}$ is:

A $-\sin\frac{x}{3}$ B $-9\sin\frac{x}{3}$ C $\sin\frac{x}{3}$ D $9\sin\frac{x}{3}$

$\int 3\cos\frac{x}{3}\, dx = 3\int \cos\left(\frac{x}{3}\right) dx = 3\sin\left(\frac{x}{3}\right) \times 3 + C = 9\sin\left(\frac{x}{3}\right) + C$

$\boxed{D}$

$\sin \quad \cos$
$\cos \quad -\sin$

PRIMITIVES OF TRIGONOMETRIC FUNCTIONS

3 Find:

(a) $\int\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)dx$ (b) $\int(\sin x-\cos 2x)dx$ (c) $\int \sin\left(2x+\frac{\pi}{2}\right)dx$

a) $\int\left[\sin\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)\right]dx = \left[\sin\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)\right]x + C = \left[\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right]x + C = \sqrt{2}x + C$

b) $\int(\sin x - \cos 2x)\,dx = -\cos x - \frac{\sin 2x}{2} + C$

c) $\int \sin\left(2x+\frac{\pi}{2}\right)dx = -\cos\left(2x+\frac{\pi}{2}\right) \times \frac{1}{2} + C = -\frac{1}{2}\cos\left(2x+\frac{\pi}{2}\right)+C$

which is equal to $+\frac{1}{2}\sin 2x + C$

So $\int \sin\left(2x+\frac{\pi}{2}\right)dx = \frac{1}{2}\sin 2x + C$

(d) $\int \cos\left(2x-\frac{\pi}{4}\right)dx$ (e) $\int \sec^2 3x\,dx$ (f) $\int\left(\frac{1}{2}\sin 2x-\cos x\right)dx$

d) $\int \cos\left(2x-\frac{\pi}{4}\right)dx = \sin\left(2x-\frac{\pi}{4}\right)\times\frac{1}{2} + C$

e) $\int \sec^2 3x\,dx = \frac{\tan 3x}{3} + C$

f) $\int\left(\frac{1}{2}\sin 2x - \cos x\right)dx = \frac{1}{2}\int \sin 2x\,dx - \int \cos x\,dx$

$= \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) - \sin x + C$

$= -\frac{\cos 2x}{4} - \sin x + C$

4 (a) Differentiate $f(x) = log_e(sin\ x)$ (b) Hence integrate $\frac{cos\ x}{sin\ x}$

a) we know that $(\ln x)' = \frac{1}{x}$, so using the Chain rule:

$$f'(x) = \frac{1}{\sin x} \times (\sin x)' = \frac{\cos x}{\sin x}$$

b) $$\int \frac{\cos x}{\sin x}\,dx = \int \frac{d}{dx}\left(\ln(\sin x)\right)dx = \ln(\sin x) + C$$

NOTE: in fact it's $\ln|\sin x| + C$ as $\ln x$ does not take negative values (see next lesson)

5 The gradient of a curve is given by $\frac{dy}{dx} = 2\sin 3x$. If the curve passes through the point $\left(\frac{\pi}{3}, 3\right)$, find the equation of the curve.

$$y = \int 2\sin 3x\,dx = 2\int \sin 3x\,dx = 2\left(\frac{-\cos 3x}{3}\right) + C$$

So $y = -\frac{2}{3}\cos 3x + C$

At $x = \frac{\pi}{3}$, $y = 3$ so $3 = -\frac{2}{3}\cos 3 \times \frac{\pi}{3} + C$

$\therefore$ $C = 3 + \frac{2}{3}\cos\pi = 3 - \frac{2}{3} = \frac{7}{3}$

So the equation of the curve is $y = -\frac{2}{3}\cos 3x + \frac{7}{3}$