# THE SECOND DERIVATIVE AND TURNING POINTS

**1** For $y = 2x^3 + 3x^2 - 12x + 2$, find any stationary points and determine their nature. Sketch the curve, showing the turning points and any points of inflection.

**3** Find the local maxima, minima and points of inflection of $f(x) = x^2(3 - x)$. Sketch the graph of $f$.

**5** A function $f(x)$ is defined by $y = x^3(x - 2)$.

**(a)** Find the coordinates of the turning points of $y = f(x)$.

**(b)** Find the coordinates of the points of inflection.

**(c)** Hence sketch the graph of $y = f(x)$, showing the turning point, the points of inflection and the points where the curve meets the $x$-axis.

**(d)** What is the minimum value of $f(x)$ for $-1 \leq x \leq 3$?

**8** Let $f(x) = x^4 - x^2$.

**(a)** Find the coordinates of the points where the curve crosses the axes.

**(b)** Find the coordinates of the stationary points and determine their nature.

**(c)** Find the coordinates of the points of inflection.

**(d)** Sketch the graph of $y = f(x)$ for $-1.5 \leq x \leq 1.5$, indicating clearly the intercepts, stationary points and points of inflection.

**(e)** For what values of $x$ is the curve concave down?

**13** The revenue function for a magazine is given by $R = 4500x - 500x^2$, where $x$ is the cost per issue of the magazine. What will be the cost per issue of the magazine to achieve maximum revenue?

**14** The revenue equation for a manufacturer is $R = \frac{80x - x^2}{4}$, where $x$ is the number of units sold. How many units must be sold to achieve maximum revenue?

**15** A supplier has a monopoly on sales of books. The supplier's profit function is given by $P = 396x - 2.2x^2 - 400$, where $x$ is the number of books sold.

**(a)** How many books must the supplier sell to maximise the profit?

**(b)** What is the maximum profit?

**(c)** If the government imposes a new 'monopoly tax' of \$22 per book on the supplier, what is the new profit equation?

**(d)** Under the monopoly tax, how many books must the supplier now sell to maximise the profit? What is the new maximum profit?