ABSOLUTE VALUE FUNCTIONS

Given the graph of $y = f(x)$, it is often useful or necessary to draw the graphs of the absolute value functions $y = |f(x)|$ and $y = f(|x|)$. Sometimes these graphs will be the same, but not always.

$y = |f(x)|$ is defined wherever $f(x)$ exists. $|f(x)| \geq 0$ if $-\infty < f(x) < \infty$. $|f(x)| > 0$ wherever $f(x) < 0$.

$y = f(|x|)$ is the same as $y = f(x)$ for $x \geq 0$, but different for $x < 0$.

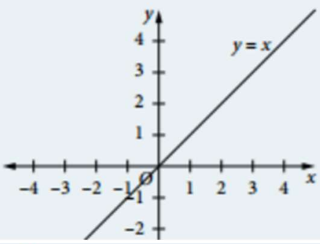
Example 7

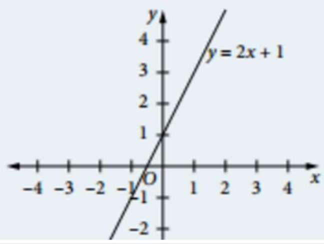
In each part, use the graph of the given function to draw the graph of $y = |f(x)|$ and the graph of $y = f(|x|)$.

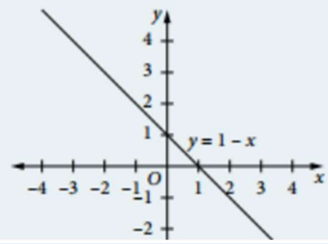
(a) Given the graph of $y = x$, draw $y = |x|$.

(b) Given the graph of $y = 2x + 1$, draw $y = |2x + 1|$ and $y = 2|x| + 1$.

(c) Given the graph of $y = 1 - x$, draw $y = |1 - x|$ and $y = 1 - |x|$.

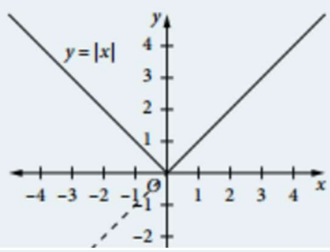






Solution

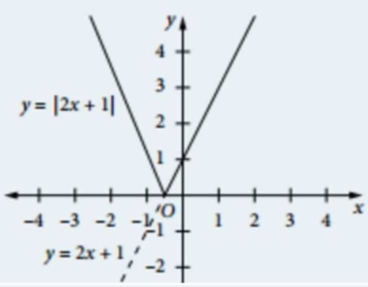
(a) The graphs are the same for $x \geq 0$.
$|x| \geq 0$ for all x.

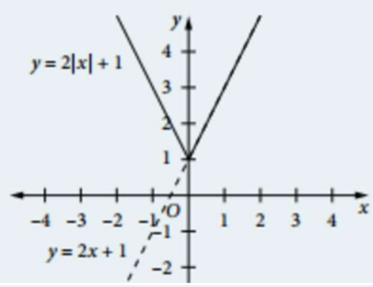


(b) The graphs of $y = 2x + 1$ and $y = |2x + 1|$ are the same for $x \geq -\frac{1}{2}$.
$|2x + 1| \geq 0$ for all x.

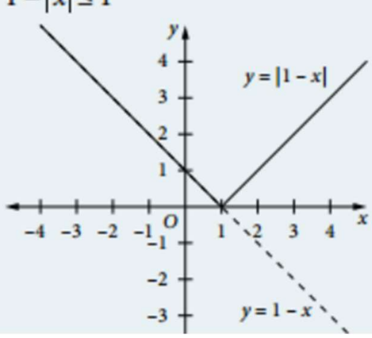
The graphs of $y = 2x + 1$ and $y = 2|x| + 1$ are the same for $x \geq 0$.

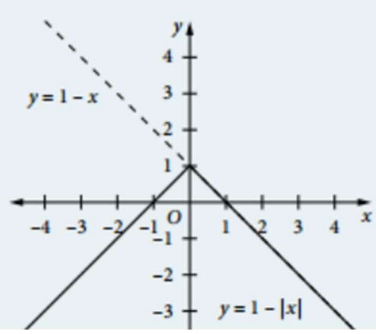
$|2x + 1| \geq 1$ for all x.





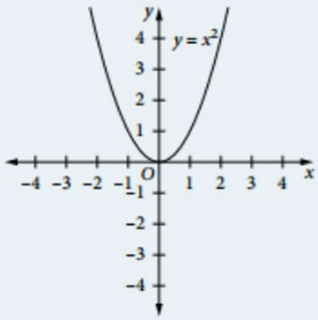
(c) The graphs of $y = 1 - x$ and $y = |1 - x|$ are the same for $x \leq 1$.
$|1 - x| \geq 0$ for all x.
The graphs of $y = 1 - x$ and $y = 1 - |x|$ are the same for $x \geq 0$.
$1 - |x| \leq 1$



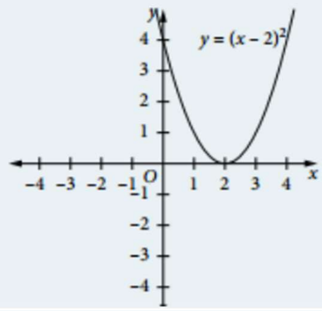


Example 8

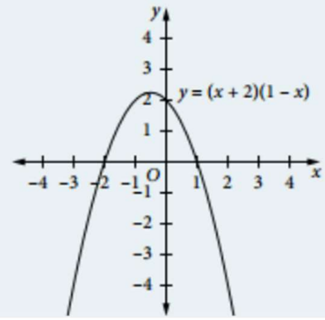
(a) Given the graph of $y = x^2$, draw $y = |x^2|$ and $y = |x|^2$.



(b) Given the graph of $y = (x-2)^2$, draw $y = |(x-2)^2|$ and $y = (|x|-2)^2$.



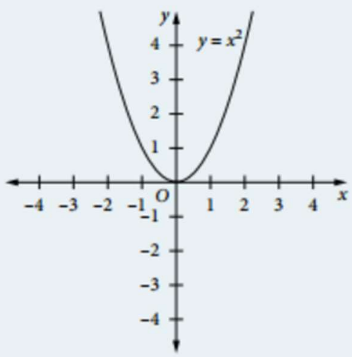
(c) Given the graph of $y = (x+2)(1-x)$, draw $y = |(x+2)(1-x)|$ and $y = (|x|+2)(1-|x|)$.



Solution

(a) The graphs of $y = |x^2|$ and $y = |x|^2$ are the same as the original graph $y = x^2$.

By the definition, $|x| = \sqrt{x^2}$ so $|x|^2 = x^2$.

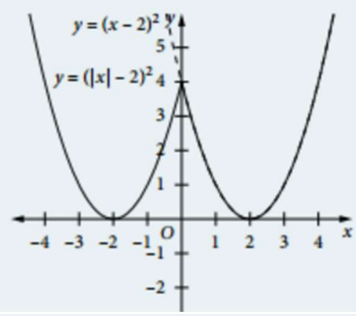


(b) The graphs of $y = (x-2)^2$ and $y = |(x-2)^2|$ are the same.

The graphs of $y = (x-2)^2$ and $y = (|x|-2)^2$ are the same for $x \geq 0$.

There is a cusp at (0, 4). $x = 0$ is an axis of symmetry.

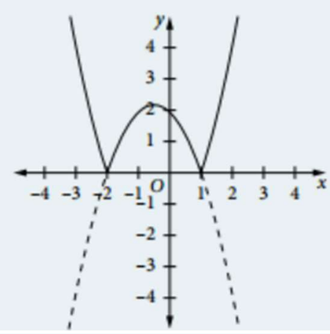
$(|x|-2)^2 \geq 0$



(c) The graphs of $y = (x+2)(1-x)$ and $y = |(x+2)(1-x)|$ are the same for $-2 \leq x \leq 1$.

There is a cusp at (−2, 0) and (1, 0).

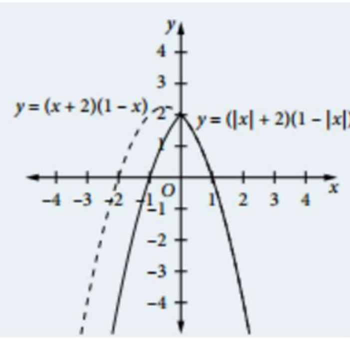
$x = -\frac{1}{2}$ is an axis of symmetry.



The graphs of $y = (x+2)(1-x)$ and $y = (|x|+2)(1-|x|)$ are the same for $x \geq 0$.

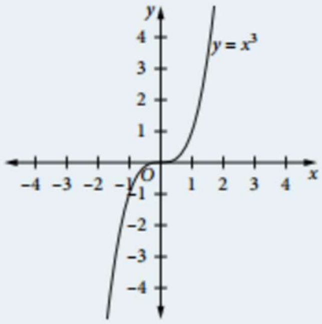
$(|x|+2)(1-|x|) \leq 2$. (0, 2) is not a turning point.

$x = 0$ is an axis of symmetry.

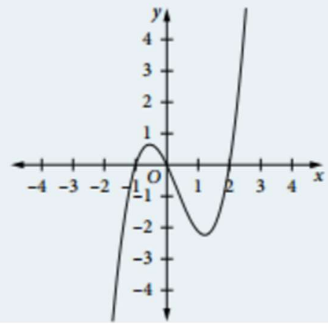


Example 9

(a) Given the graph of $y = x^3$, draw $y = |x^3|$ and $y = |x|^3$.



(b) Given the graph of $y = x(x+1)(x-2)$, draw $y = |x(x+1)(x-2)|$ and $y = |x|(|x|+1)(|x|-2)$.

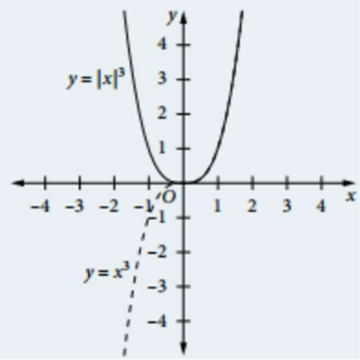


Solution

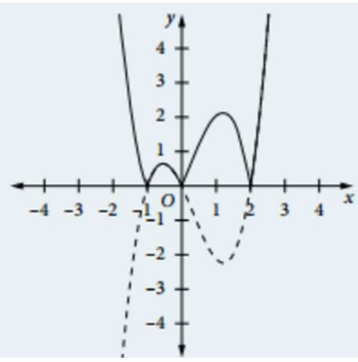
(a) $x^3 = |x^3|$ for $x \geq 0$, i.e. the curves are identical for $x \geq 0$.

Also $x^3 = |x|^3$ for $x \geq 0$, i.e. the curves are identical for $x \geq 0$.

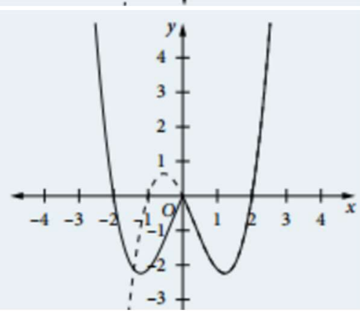
The graph of $y = |x|^3$ is the same as the graph of $y = |x^3|$.
They are both the solid line in the graph.



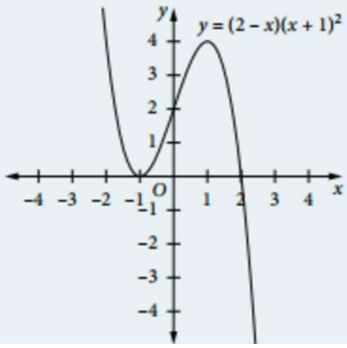
(b) For $y = |x(x+1)(x-2)|$:
The graphs meet at $(-1, 0)$, $(0, 0)$, $(2, 0)$.
The graphs are the same for $-1 \leq x \leq 0$ and $x \geq 2$.
$|x(x+1)(x-2)| \geq 0$ for all x.



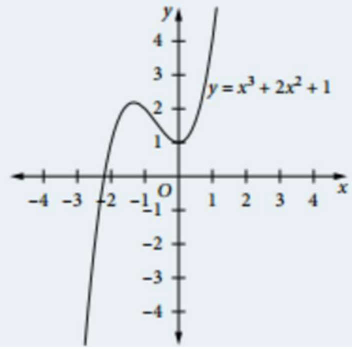
For $y = |x|(|x|+1)(|x|-2)$:
The graphs meet at $(0, 0)$, $(2, 0)$ and $(-\sqrt{2}, -2)$. This last point can be solved using graphing software.
The graphs are the same for $x \geq 0$. There is a cusp at $(0, 0)$.
$|x|(|x|+1)(|x|-2) \geq -2$.



(c) Given the graph of $y=(2-x)(x+1)^2$, draw $y=\left|(2-x)(x+1)^2\right|$ and $y=\left(2-|x|\right)\left(|x|+1\right)^2$.



(d) Given the graph of $y=x^3+2x^2+1$, draw $y=\left|x^3+2x^2+1\right|$ and $y=|x|^3+2|x|^2+1$.

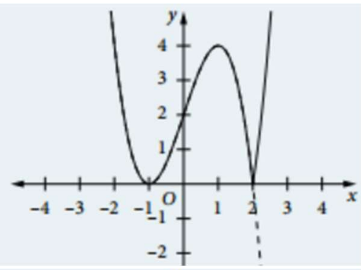


(c) For $y=\left|(2-x)(x+1)^2\right|$:

The graphs are the same for $x \le 2$.

$\left|(2-x)(x+1)^2\right| \ge 0$. (2, 0) is a cusp.

(−1, 0) is a minimum turning point, (1, 4) is a maximum turning point.



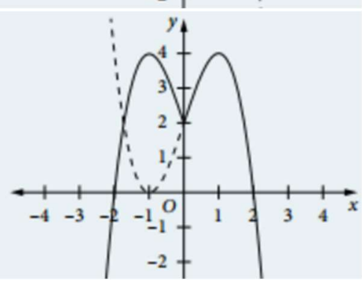
For $y=\left(2-|x|\right)\left(|x|+1\right)^2$:

The graphs are the same for $x \ge 0$.

The curves also intersect at $\left(-\sqrt{3}, 2\right)$.

(0, 2) is a cusp. (−1, 4) and (1, 4) are maximum turning points.

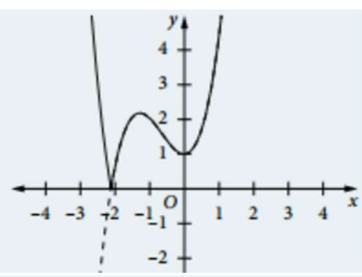
$\left(2-|x|\right)\left(|x|+1\right)^2 \le 4$



(d) For $y=\left|x^3+2x^2+1\right|$:

The graphs are the same for about $x \ge -2.2$.

(−2.2, 0) is a cusp. (0, 1) is a minimum turning point. (−1.33, 2.19) is a maximum turning point.



For $y=|x|^3+2|x|^2+1$:

The graphs are the same for $x \ge 0$.

$|x|^3+2|x|^2+1 \ge 1$

(0, 1) is a minimum turning point.