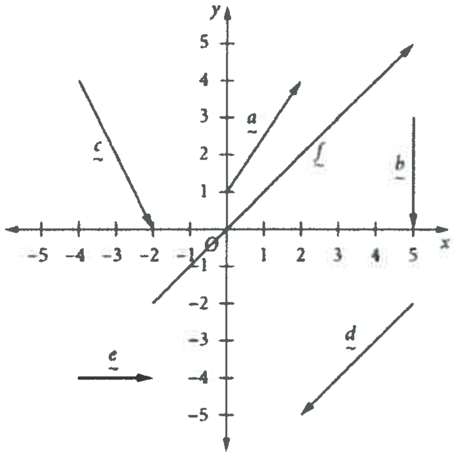
VECTORS IN COMPONENT FORM

1 Express each vector shown in component form.

(a) $\underset{\sim}{a}$ (b) $\underset{\sim}{b}$ (c) $\underset{\sim}{c}$

(d) $\underset{\sim}{d}$ (e) $\underset{\sim}{e}$ (f) $\underset{\sim}{f}$



a) $\vec{a} = 2\vec{i} + 3\vec{j}$ b) $\vec{b} = -3\vec{j}$

c) $\vec{c} = 2\vec{i} - 4\vec{j}$ d) $\vec{d} = -3\vec{i} - 3\vec{j}$

e) $\vec{e} = 2\vec{i}$ f) $\vec{f} = 7\vec{i} + 7\vec{j}$

2 Find the magnitude of the following vectors.

(a) $\underset{\sim}{a} = 5\underset{\sim}{i} + 4\underset{\sim}{j}$ (b) $-4\underset{\sim}{i} + 7\underset{\sim}{j}$ (c) $7\underset{\sim}{i} - 24\underset{\sim}{j}$ (d) $-5\underset{\sim}{i}$

a) $|\underset{\sim}{a}| = \sqrt{5^2+4^2} = \sqrt{41}$ b) $|\vec{a}| = \sqrt{4^2+7^2} = \sqrt{65}$

c) $|\underset{\sim}{a}| = \sqrt{7^2+24^2} = 25$ d) $|\vec{a}| = \sqrt{25} = 5$

3 Resolve the following vectors into component form $x\underset{\sim}{i} + y\underset{\sim}{j}$, correct to two decimal places.

(a) $\underset{\sim}{a}$ has a magnitude of 15 units and has a direction of 35° to the positive x-axis.

(b) $\underset{\sim}{b}$ has a magnitude of 23 units and has a direction of 121° to the positive x-axis.

a) $\underset{\sim}{a} = 15\cos 35\,\vec{i} + 15\sin 35\,\vec{j} = 12.29\,\vec{i} + 8.60\,\vec{j}$

b) $\underset{\sim}{b} = 23\cos 121\,\vec{i} + 23\sin 121\,\vec{j} = -11.85\,\vec{i} + 19.71\,\vec{j}$

4 Given $\underset{\sim}{a} = 4\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$, find: (a) $\underset{\sim}{a} + \underset{\sim}{b}$ (b) $\underset{\sim}{b} - \underset{\sim}{a}$ (c) $2\underset{\sim}{a} + 7\underset{\sim}{b}$

a) $\vec{a} + \vec{b} = 7\vec{i} - 3\vec{j}$

b) $\vec{b} - \vec{a} = -\vec{i} + 7\vec{j}$

c) $2\vec{a} + 7\vec{b} = 8\vec{i} - 10\vec{j} + 21\vec{i} + 14\vec{j}$
$= 29\vec{i} + 4\vec{j}$

6 Find the values of the unknown pronumerals in the following equations.

(a) $5\underset{\sim}{i} - 4\underset{\sim}{j} = 3a\underset{\sim}{i} + 2b\underset{\sim}{j}$

$3a = 5$ so $a = 5/3$

$2b = -4$ so $b = -2$

(b) $(x+2y)\underset{\sim}{i} + y\underset{\sim}{j} = -3\underset{\sim}{i} + 7\underset{\sim}{j}$

$\begin{cases} x + 2y = -3 \\ y = 7 \end{cases}$ so $x = -3 - 2xy$

$x = -3 - 2 \times 7 = -17$

(e) $(x^2 + 5x)\underset{\sim}{i} + (y^3 - 1)\underset{\sim}{j} = -6\underset{\sim}{i} + 7\underset{\sim}{j}$

$\begin{cases} x^2 + 5x = -6 \\ y^3 - 1 = 7 \end{cases}$ so $x^2 + 5x + 6 = 0$; $\Rightarrow y = 2$

$\Delta = 25 - 4 \times 6 = 1$

$x = \dfrac{-5 \pm 1}{2}$ so $x = -3$ or $x = -2$

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10 Given $\underset{\sim}{a} = -13\underset{\sim}{i} + 20\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 15\underset{\sim}{j}$, find:

(a) $|\underset{\sim}{a} - \underset{\sim}{b}|$ **(b)** the value of x so that the vector $x\underset{\sim}{a} + 4\underset{\sim}{b}$ is parallel to the x-axis.

a) $|\underset{\sim}{a} - \underset{\sim}{b}| = |-13\vec{i} + 20\vec{j} - (2\vec{i} + 15\vec{j})| = |-15\vec{i} + 5\vec{j}| = \sqrt{15^2 + 5^2} = \sqrt{250} = 5\sqrt{10}$

b) $x\vec{a} + 4\vec{b} = x(-13\vec{i} + 20\vec{j}) + 4(2\vec{i} + 15\vec{j}) = \vec{i}(-13x + 8) + \vec{j}(20x + 60)$

For $(x\vec{a} + 4\vec{b})$ to be parallel to the x-axis, we must have $20x + 60 = 0$

so $x = -3$

12 Which one of the following vectors is parallel to the vector $\underset{\sim}{f} = 14\underset{\sim}{i} - 6\underset{\sim}{j}$?

A $\underset{\sim}{a} = 28\underset{\sim}{i} + 12\underset{\sim}{j}$	**B** $\underset{\sim}{b} = 14\underset{\sim}{i} + 6\underset{\sim}{j}$	**C** $\underset{\sim}{c} = -14\underset{\sim}{i} - 6\underset{\sim}{j}$	**D** $\underset{\sim}{d} = -28\underset{\sim}{i} + 12\underset{\sim}{j}$
NO	NO	NO	YES,

as $\vec{d} = -2 \times \vec{f}$

16 For $\underset{\sim}{b} = 3\underset{\sim}{i} - 9\underset{\sim}{j}$:

(a) find $\hat{\underset{\sim}{b}}$ **(b)** find vector $\underset{\sim}{c}$ in the direction of $\underset{\sim}{b}$ with a magnitude of 15.

a) $|\vec{b}| = \sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$ so $\hat{\underset{\sim}{b}} = \frac{1}{3\sqrt{10}}(3\vec{i} - 9\vec{j}) = \frac{1}{\sqrt{10}}(\vec{i} - 3\vec{j})$

b) $\vec{c} = 15\hat{\underset{\sim}{b}} = 15 \times \frac{1}{\sqrt{10}}(\vec{i} - 3\vec{j}) = \frac{15}{\sqrt{10}}(\vec{i} - 3\vec{j})$ $= \frac{\sqrt{10}}{10}(\vec{i} - 3\vec{j})$

$\vec{c} = \frac{15\sqrt{10}}{10}(\vec{i} - 3\vec{j}) = \frac{3\sqrt{10}}{2}(\vec{i} - 3\vec{j})$

⚠ NOTE: in a test / HSC, always rationalise the denominator of fractions.

19 What is the unit vector in the direction of $\underset{\sim}{a} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$ is?

A $\frac{1}{7}(-2\underset{\sim}{i} + 5\underset{\sim}{i})$	**B** $\frac{1}{29}(-2\underset{\sim}{i} + 5\underset{\sim}{j})$	**C** $\frac{1}{\sqrt{29}}(-2\underset{\sim}{i} + 5\underset{\sim}{j})$	**D** $\frac{1}{\sqrt{21}}(-2\underset{\sim}{i} + 5\underset{\sim}{j})$

$|\underset{\sim}{a}| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$

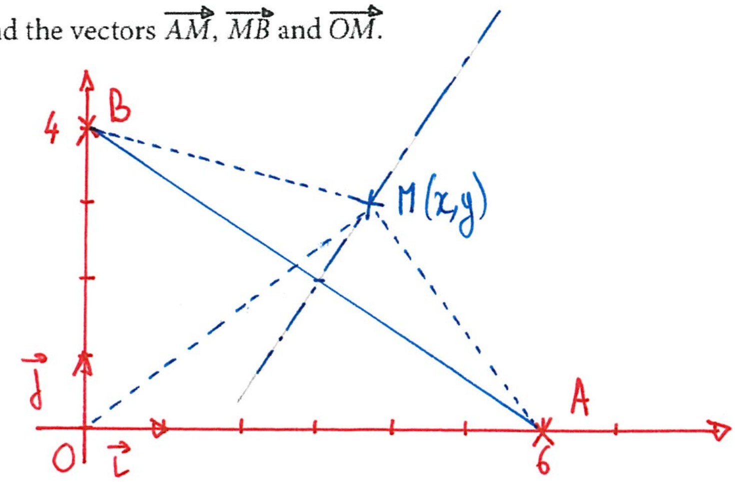
so $\boxed{C}$

23 ΔOAB is a triangle in which $\overrightarrow{OA} = 6\underset{\sim}{i}$ and $\overrightarrow{OB} = 4\underset{\sim}{j}$. The point M with position vector $\overrightarrow{OM} = x\underset{\sim}{i} + y\underset{\sim}{j}$ is equidistant from O, A and B.

(a) Find the values of x and y.

(b) Find the vectors $\overrightarrow{AM}$, $\overrightarrow{MB}$ and $\overrightarrow{OM}$.

(c) Find the values of $|\overrightarrow{AM}|$, $|\overrightarrow{MB}|$ and $|\overrightarrow{OM}|$.



a) We know that

$|\overrightarrow{OM}| = |\overrightarrow{MB}| = |\overrightarrow{MA}|$, so:

$|\overrightarrow{OM}| = |\overrightarrow{MB}|$

$\iff \sqrt{x^2+y^2} = \sqrt{x^2+(y-4)^2}$

$\iff \cancel{x^2}+y^2 = \cancel{x^2} + (y-4)^2$

$\iff \cancel{y^2} = \cancel{y^2} - 8y + 16$

$\iff \boxed{y = 2}$

From $|\overrightarrow{OM}| = |\overrightarrow{MA}|$, we get:

$\sqrt{x^2+y^2} = \sqrt{(x-6)^2+y^2}$

$\iff x^2 + \cancel{y^2} = (x-6)^2 + \cancel{y^2}$

$\iff \cancel{x^2} = \cancel{x^2} - 12x + 36$

$\implies \boxed{x = 3}$

So $M(3,2)$

M is actually the midpoint of AB.

b) $\overrightarrow{AM} = -3\vec{i} + 2\vec{j}$

$\overrightarrow{MB} = -3\vec{i} + 2\vec{j}$

$\overrightarrow{OM} = 3\vec{i} + 2\vec{j}$

c) $|\overrightarrow{AM}| = \sqrt{(-3)^2+2^2} = \sqrt{13}$

$|\overrightarrow{MB}| = \sqrt{(-3)^2+2^2} = \sqrt{13}$

$|\overrightarrow{OM}| = \sqrt{3^2+2^2} = \sqrt{13}$

So M is actually the centre of the circle passing through O, A and B

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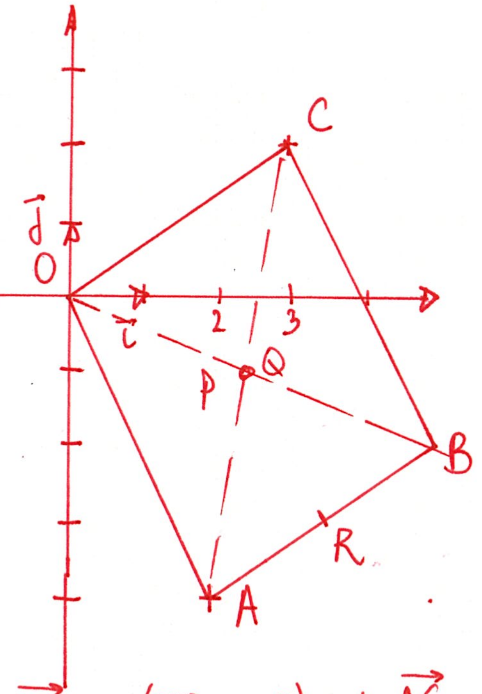
24 $OABC$ is a parallelogram in which vectors $\overrightarrow{OA} = 2\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\overrightarrow{OC} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$.

(a) Find vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

(b) Find the vectors $\overrightarrow{OB}$ and $\overrightarrow{AC}$, the diagonals of the parallelogram.

(c) Find the vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, where P is the midpoint of $\overline{OB}$ and Q is the midpoint of $\overline{AC}$. What can you say about the points P and Q?

(d) Find the vectors $\overrightarrow{OR}$ and $\overrightarrow{CR}$, where R is the midpoint of $\overline{AB}$.



a) $\overrightarrow{AB} = \overrightarrow{OC} = 3\vec{i} + 2\vec{j}$

$\overrightarrow{CB} = \overrightarrow{OA} = 2\vec{i} - 4\vec{j}$

b) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 2\vec{i} - 4\vec{j} + 3\vec{i} + 2\vec{j}$

So $\overrightarrow{OB} = 5\vec{i} - 2\vec{j}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -(2\vec{i} - 4\vec{j}) + 3\vec{i} + 2\vec{j}$

So $\overrightarrow{AC} = \vec{i} + 6\vec{j}$

c) $\overrightarrow{OP} = \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}(5\vec{i} - 2\vec{j}) = \frac{5}{2}\vec{i} - \vec{j}$

whereas $\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ} = (3\vec{i} + 2\vec{j}) + \frac{1}{2}\overrightarrow{CA} = (3\vec{i} + 2\vec{j}) - \frac{1}{2}\overrightarrow{AC}$

So $\overrightarrow{OQ} = 3\vec{i} + 2\vec{j} - \frac{1}{2}(\vec{i} + 6\vec{j}) = \frac{5}{2}\vec{i} - \vec{j}$

So $\overrightarrow{OP} = \overrightarrow{OQ}$ $\therefore$ P and Q are the same point, as the 2 vectors have the same tail. So the diagonals of the //gram bisect each other.

d) $\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR} = (2\vec{i} - 4\vec{j}) + \frac{1}{2}\overrightarrow{AB} = (2\vec{i} - 4\vec{j}) + \frac{1}{2}(3\vec{i} + 2\vec{j})$

So $\overrightarrow{OR} = \frac{7}{2}\vec{i} - 3\vec{j}$

$\overrightarrow{CR} = \overrightarrow{CB} + \overrightarrow{BR} = 2\vec{i} - 4\vec{j} - \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{CR} = 2\vec{i} - 4\vec{j} - \frac{1}{2}(3\vec{i} + 2\vec{j})$

$\overrightarrow{CR} = \frac{1}{2}\vec{i} - 5\vec{j}$

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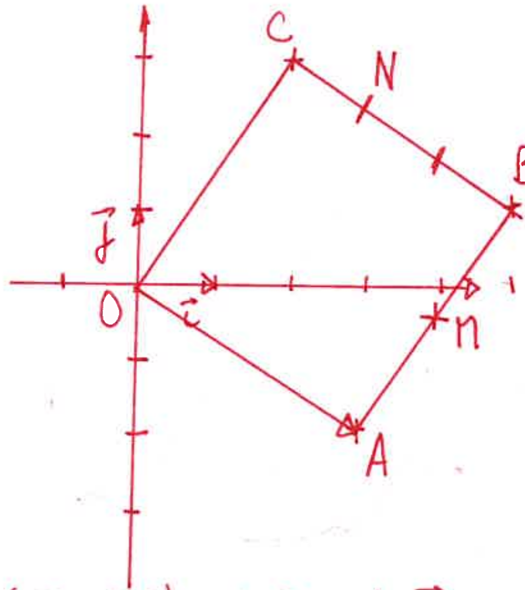
25 $OABC$ is a square in which vectors $\overrightarrow{OA} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\overrightarrow{OC} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$. M is the midpoint of $\overline{AB}$ and N divides $\overline{CB}$ internally in the ratio $1:2$.

(a) Find the vectors $\overrightarrow{OB}$, $\overrightarrow{AC}$, $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{NB}$. **(b)** Find the length of the diagonals, $|\overrightarrow{OB}|$ and $|\overrightarrow{AC}|$.



a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC}$

$\overrightarrow{OB} = (3\vec{i} - 2\vec{j}) + (2\vec{i} + 3\vec{j})$

$\overrightarrow{OB} = 5\vec{i} + \vec{j}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\overrightarrow{OA} + \overrightarrow{OC}$

$\overrightarrow{AC} = -(3\vec{i} - 2\vec{j}) + (2\vec{i} + 3\vec{j})$

$\overrightarrow{AC} = -\vec{i} + 5\vec{j}$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = 3\vec{i} - 2\vec{j} + \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{OM} = 3\vec{i} - 2\vec{j} + \frac{1}{2}\overrightarrow{OC} = 3\vec{i} - 2\vec{j} + \frac{1}{2}(2\vec{i} + 3\vec{j}) = 4\vec{i} - \frac{1}{2}\vec{j}$

$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = 2\vec{i} + 3\vec{j} + \frac{1}{3}\overrightarrow{CB} = 2\vec{i} + 3\vec{j} + \frac{1}{3}\overrightarrow{OA}$

So $\overrightarrow{ON} = 2\vec{i} + 3\vec{j} + \frac{1}{3}(3\vec{i} - 2\vec{j}) = 3\vec{i} + \frac{7}{3}\vec{j}$

$\overrightarrow{NB} = \frac{2}{3}\overrightarrow{CB} = \frac{2}{3}\overrightarrow{OA} = \frac{2}{3}(3\vec{i} - 2\vec{j}) = 2\vec{i} - \frac{4}{3}\vec{j}$

b) $\overrightarrow{OB} = 5\vec{i} + \vec{j}$ so $|\overrightarrow{OB}| = \sqrt{25+1} = \sqrt{26}$

and of course $|\overrightarrow{AC}| = \sqrt{26}$ as it's a square so the diagonals have the same length.

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27 **(a)** If $\underset{\sim}{a} = 3p\underset{\sim}{i} + 4p\underset{\sim}{j}$, $p > 0$ and $|\underset{\sim}{a}| = 2$, find the exact value of p. **(b)** Hence find $\hat{\underset{\sim}{a}}$.

(c) Find the vector $\underset{\sim}{b}$ which is parallel to $\hat{\underset{\sim}{a}}$, if $|\underset{\sim}{b}| = 10$.

(d) If $\underset{\sim}{c} = 7q\underset{\sim}{i} + 24q\underset{\sim}{j}$, $q > 0$, and $|\underset{\sim}{c}| = 4$, find the exact value of q. **(e)** Hence find $\hat{\underset{\sim}{c}}$.

(f) Find the vector $\underset{\sim}{d}$ in the direction of $\hat{\underset{\sim}{c}}$ where $|\underset{\sim}{d}| = 50$.

(g) Find the vector with magnitude 10 that is parallel to the vector $\underset{\sim}{b} + \underset{\sim}{d}$.

a) $|\underset{\sim}{a}| = \sqrt{9p^2 + 16p^2} = \sqrt{25p^2} = 5p$ but $|\underset{\sim}{a}| = 2$ so $p = 2/5$

b) $\hat{\underset{\sim}{a}} = \frac{1}{2}\left[\frac{6}{5}\vec{i} + \frac{8}{5}\vec{j}\right] = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

c) $\underset{\sim}{b} = 10\,\hat{\underset{\sim}{a}} = 10 \times \left[\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}\right] = 6\vec{i} + 8\vec{j}$

d) $|\underset{\sim}{c}| = \sqrt{49q^2 + 576q^2} = 25q$ but $|\underset{\sim}{c}| = 4$ so $q = \frac{4}{25}$

e) $\hat{\underset{\sim}{c}} = \frac{1}{4}\left[\frac{28}{25}\vec{i} + \frac{96}{25}\vec{j}\right] = \frac{7}{25}\vec{i} + \frac{24}{25}\vec{j}$

f) $\underset{\sim}{d} = 50 \times \hat{\underset{\sim}{c}} = 50 \times \left[\frac{7}{25}\vec{i} + \frac{24}{25}\vec{j}\right]$

$\underset{\sim}{d} = 14\vec{i} + 48\vec{j}$

g) $\underset{\sim}{b} + \underset{\sim}{d} = 6\vec{i} + 8\vec{j} + 14\vec{i} + 48\vec{j} = 20\vec{i} + 56\vec{j}$

$|\underset{\sim}{b} + \underset{\sim}{d}| = \sqrt{400 + 3136} = \sqrt{3536} = 4\sqrt{221}$

So if $\underset{\sim}{u} = \underset{\sim}{b} + \underset{\sim}{d}$ then $\hat{\underset{\sim}{u}} = \frac{1}{4\sqrt{221}}\left[20\vec{i} + 56\vec{j}\right]$

and the vector $\vec{t}$ of magnitude 10 in the direction of $\vec{u}$

would be : $\vec{t} = 10 \times \frac{1}{4\sqrt{221}}\left[20\vec{i} + 56\vec{j}\right]$

$\vec{t} = \frac{1}{\sqrt{221}}\left[50\vec{i} + 140\vec{j}\right] = \frac{10\sqrt{221}}{221}\left[5\vec{i} + 14\vec{j}\right]$