OTHER USEFUL TECHNIQUES

1 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx = \ldots$

A $2\int_{0}^{\frac{\pi}{2}} x\cos x\,dx$ **B** 0 **C** $\pi - 2$ **D** $\frac{\pi}{2} - 1$

2 Evaluate $\int_{0}^{\frac{1}{2}} \sqrt{\frac{1+x}{1-x}}\,dx$ using: $\frac{1+x}{1-x} = \frac{(1+x)^2}{1-x^2}$

OTHER USEFUL TECHNIQUES

3 **(a)** Write $(x-1)(3-x)$ in the form $b^2-(x-a)^2$ where a and b are real numbers.

(b) Using the values of a and b from part **(a)** and making the substitution $x-a=b\sin\theta$, or otherwise, evaluate: $\int_1^3 \sqrt{(x-1)(3-x)}\,dx$

OTHER USEFUL TECHNIQUES

4 **(a)** Use the substitution $t = \tan \frac{x}{2}$ to find: $\int \sec x \, dx$

(b) By rewriting $\sec x$ as $\frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$, find: $\int \sec x \, dx$

5 **(a)** Find: $\int \sec x \tan x \, dx$

(b) Using the substitution $u = \frac{\pi}{2} - x$, find: $\int \operatorname{cosec} x \cot x \, dx$

6 Find the length of the circumference of the circle $x^2 + y^2 = r^2$ using the arc length formula on the circle's first quadrant, i.e. arc length $= \int_0^r \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$.

7 Let $I = \int_1^2 \frac{\cos^2\left(\frac{\pi}{6}x\right)}{x(3-x)} dx$.

(a) Use the substitution $u = 3 - x$ to show that: $I = \int_1^2 \frac{\sin^2\left(\frac{\pi}{6}u\right)}{u(3-u)} du$

(b) Hence find the value of I.

OTHER USEFUL TECHNIQUES

8 **(a)** Differentiate $x^2 \tan^{-1} x$ with respect to x. **(b)** Hence find: $\int 2x \tan^{-1} x \, dx$

OTHER USEFUL TECHNIQUES

9 Show that: **(a)** $\int_{-1}^{1} x^3(1-x^2)^2\,dx = 0$ **(b)** $\int_{-1}^{1} x^2(1-x^2)^3\,dx = 2\int_{0}^{1} x^2(1-x^2)^3\,dx$

10 **(a)** By using an appropriate substitution, show that: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(b) Hence show that: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\,dx$

12 **(a)** If $x > 0$ and $1 < u < 1 + x$, show that: $\frac{1}{1+x} < \frac{1}{u} < 1$

(b) By integrating each term of $\frac{1}{1+x} < \frac{1}{u} < 1$ with respect to u between 1 and $(1 + x)$, show that: $\frac{x}{1+x} < \log_e(1+x) < x$