1 Convert to degrees: $\left(\times \frac{180}{\pi}\right)$

a $\frac{\pi}{5} = 36°$ **b** $\frac{2\pi}{3} = 120°$ **c** $\frac{5\pi}{4} = 225°$ **d** $\frac{7\pi}{6} = 210°$ **e** $3\pi = 540°$

f $\frac{7\pi}{9} = 140°$ **g** $\frac{4\pi}{3} = 240°$ **h** $\frac{7\pi}{3} = 420°$ **i** $\frac{\pi}{9} = 20°$ **j** $\frac{5\pi}{18} = 50°$

2 Convert to radians in terms of π: $\left(\times \frac{\pi}{180}\right)$

a $135° = \frac{3\pi}{4}$ **b** $30° = \frac{\pi}{6}$ **c** $150° = \frac{5\pi}{6}$ **d** $240° = \frac{4\pi}{3}$ $\frac{3\pi}{2} - \frac{\pi}{6}$ **e** $300° = \frac{5\pi}{3}$

f $63° = \frac{7\pi}{20}$ **g** $15° = \frac{\pi}{12}$ **h** $450° = \frac{5\pi}{2}$ **i** $225° = \frac{5}{4}\pi$ **j** $120° = \frac{2}{3}\pi$

3 Change to radians, correct to 2 decimal places: $\left(\times \frac{\pi}{180}\right)$

a $56°$ $= 0.98$ **b** $68°$ $= 1.19$ **c** $127°$ $= 2.22$ **d** $289°$ $= 5.04$ **e** $312°$ $= 5.45$

4 Change to radians, correct to 2 decimal places: $\left(\times \frac{\pi}{180}\right)$

a $18°34'$ $= 0.32$ **b** $35°12'$ $= 0.61$ **c** $101°56'$ $= 1.78$ **d** $88°29'$ $= 1.54$ **e** $50°39'$ $= 0.88$

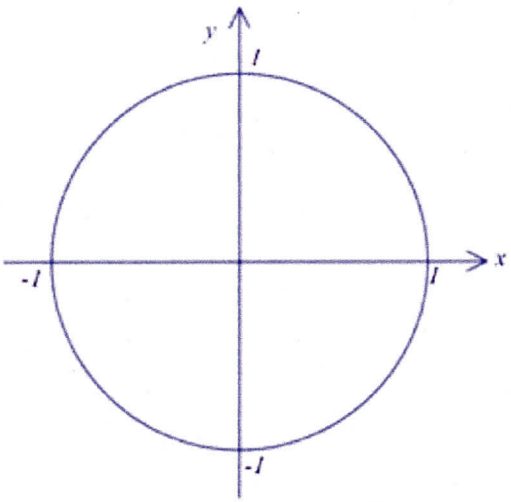
5 Convert each radian measure into degrees and minutes, to the nearest minute: $\left(\times \frac{180}{\pi}\right)$

a 1.09 $= 62°27'$ **b** 0.768 $= 44°00'$ **c** 1.16 $= 66°28'$ **d** 0.99 $= 56°43'$ **e** 0.32 $= 18°20'$

6 Find correct to 2 decimal places: SHIFT MODE 4 to convert to Radians

a $\sin 0.342$ $= 0.34$ **b** $\cos 1.5$ $= 0.07$ **c** $\tan 0.056$ $= 0.06$ **d** $\cos 0.589$ $= 0.83$ **e** $\tan 2.29$ $= -1.14$

ALL EXACT VALUES FROM ANGLES IN RADIANS



Angle in radians	Sine	Cosine	Tangent
0	0	1	0
$\frac{\pi}{6}$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\frac{\pi}{4}$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\frac{\pi}{3}$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined (∞)
$\frac{2\pi}{3}$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
$\frac{3\pi}{4}$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\sqrt{3}/2$	$-1/\sqrt{3}$
π	0	-1	0
$\frac{7\pi}{6}$	$-1/2$	$-\sqrt{3}/2$	$1/\sqrt{3}$
$\frac{5\pi}{4}$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1
$\frac{4\pi}{3}$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$
$\frac{3\pi}{2}$	-1	0	undefined (∞)
$\frac{5\pi}{3}$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$
$\frac{7\pi}{4}$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1
$\frac{11\pi}{6}$	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$
2π	0	1	0

1 Find the exact arc length of a circle with:

a radius 4 cm and angle subtended π $\quad \ell = 4\pi$ cm

b radius 3 m and angle subtended $\frac{\pi}{3}$ $\quad \ell = \frac{\pi}{3} \times 3 = \pi$ cm

c radius 10 cm and angle subtended $\frac{5\pi}{6}$ $\quad \ell = \frac{5\pi}{6} \times 10 = \frac{25}{3}\pi$ cm

2 Find the arc length, correct to 2 decimal places, given:

a radius 1.5 m and angle subtended 0.43 $\quad \ell = 1.5 \times 0.43 = 0.645$ m

b radius 3.21 cm and angle subtended 1.22 $\quad \ell = 1.22 \times 3.21 = 3.92$ m

3 The angle subtended at the centre of a circle of radius 3.4 m is 29°51′. Find the length of the arc cut off by this angle, correct to 1 decimal place.

$\ell = \frac{29°51'}{360} \times 2\pi \times 3.4 = 1.8$ m

4 The arc length when a sector of a circle is subtended by an angle of $\frac{\pi}{5}$ at the centre is $\frac{3\pi}{2}$ m. Find the radius of the circle.

$r = \frac{\ell}{\theta} = \frac{3\pi/2}{\pi/5} = \frac{3}{10} = 0.3$ m

5 The radius of a circle is 3 cm and an arc is $\frac{2\pi}{7}$ cm long. Find the angle subtended at the centre of the circle by the arc.

$\theta = \frac{\ell}{r} = \frac{2\pi/7}{3} = \frac{2\pi}{21}$

7 A circle with area 60 cm^2 has an arc 8 cm long. Find the angle that is subtended at the centre of the circle by the arc.

$\pi r^2 = 60$ so $r = \sqrt{\frac{60}{\pi}}$

$\ell = r\theta$ so $\theta = \frac{\ell}{r} = \frac{8}{\sqrt{60/\pi}} = 1.83$ rad $\approx 105°$

8 A circle with circumference 124 mm has a chord cut off it that subtends an angle of 40° at the centre. Find the length of the arc cut off by the chord.

$2\pi r = 124$ so $r = \frac{124}{2\pi} = \frac{62}{\pi}$

$\ell = 2\pi r \times \frac{40}{360} = \frac{2}{9}\pi \times \frac{62}{\pi} = \frac{124}{9}$ mm $= 13.8$ mm

1 Find the exact area of the sector of a circle whose radius is:

a 4 cm and the subtended angle is π **b** 3 m and the subtended angle is $\frac{\pi}{3}$

c 10 cm and the subtended angle is $\frac{5\pi}{6}$ **d** 3 cm and the subtended angle is 30°

$A = \frac{r^2\theta}{2}$ a) $A = \frac{4^2 \times \pi}{2} = 8\pi$ b) $\frac{\pi}{2}$ c) $\frac{10^2}{2} \times \left(\frac{5\pi}{6}\right) = \frac{125\pi}{3}$ d) $A = \pi \times 3^2 \times \frac{30}{360} = \frac{3\pi}{4}$

10 The area of a sector is $\frac{3\pi}{10}$ cm^2 and the arc length cut off by the sector is $\frac{\pi}{5}$ cm.

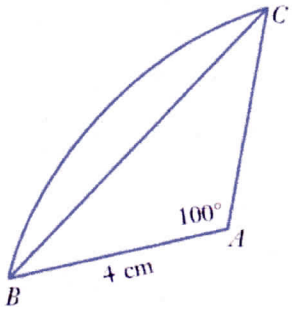
Find the angle subtended at the centre of the circle and the radius of the circle.

$A = \frac{r^2\theta}{2} = \frac{3\pi}{10}$ so $r^2\theta = \frac{3\pi}{5}$ ① then $\ell = r\theta = \frac{\pi}{5}$ ②

So, dividing ① by ②, we get $\frac{r^2\theta}{r\theta} = r = \frac{3\pi/5}{\pi/5} = 3$ cm

and $\theta = \frac{\pi}{5} \times \frac{1}{r} = \frac{\pi}{15}$

16 Arc BC subtends an angle of 100° at the centre A of a circle with radius 4 cm. Find the perimeter of sector ABC.

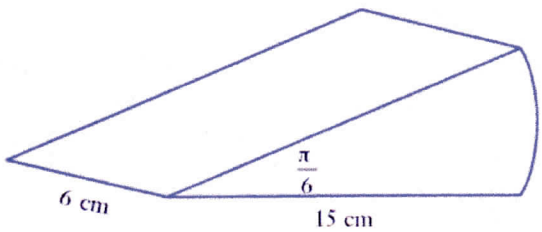


$ABC = 2 \times 4 + \overset{\frown}{BC} = 8 + \overset{\frown}{BC}$

$\overset{\frown}{BC} = \frac{100}{360} \times 2\pi \times 4 = \frac{20}{9}\pi$

So $ABC = 8 + \frac{20}{9}\pi = 14.98$ cm approx

17 A wedge is cut so that its cross-sectional area is a sector of a circle with radius 15 cm and subtending an angle of $\frac{\pi}{6}$ at the centre. Find the exact volume of the wedge.



Volume = Area of cross section × 6

$\text{Volume} = \frac{15^2 \times \pi/6}{2} \times 6$

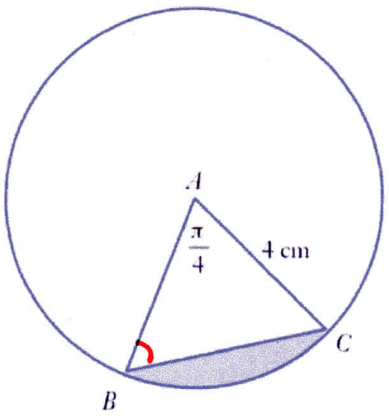
$\text{Volume} = \frac{225}{2}\pi \text{ cm}^3$

14 **a** Find the area of the sector of a circle with radius 4 cm if the angle subtended at the centre is $\frac{\pi}{4}$.

b Find the length of BC to 1 decimal place.

c Find the exact area of triangle ABC.

d Hence find the exact area of the shaded minor segment of the circle.



a) $A = \frac{r^2\theta}{2} = \frac{4^2 \times \pi/4}{2} = 2\pi \text{ cm}^2$

b) cosine rule $BC^2 = 4^2 + 4^2 - 2\times4\times4\cos\pi/4 = 32 - 32\frac{\sqrt{2}}{2} = 32 - 16\sqrt{2}$

So $BC = \sqrt{32-16\sqrt{2}} = \sqrt{2\times4^2 - 4^2\sqrt{2}} = 4\sqrt{2-\sqrt{2}}$ cm

c) $A = \frac{1}{2}bc\sin A = \frac{1}{2}\times4^2\sin\frac{\pi}{4} = 8\times\frac{\sqrt{2}}{2} = 4\sqrt{2}$ cm

d) Shaded area = $2\pi - 4\sqrt{2}$

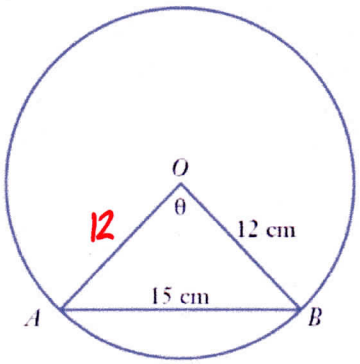
15 A triangle OAB is formed where O is the centre of a circle of radius 12 cm and A and B are endpoints of a 15 cm chord.

a Find the angle subtended at the centre of the circle, in degrees and minutes.

b Find the area of ΔOAB correct to 1 decimal place.

c Find the area of the minor segment cut off by the chord, correct to 2 decimal places.

d Find the area of the major segment cut off by the chord, correct to 2 decimal places.



a) cosine rule: $15^2 = 12^2 + 12^2 - 2\times12\times12\cos\theta$

so $\cos\theta = \frac{15^2 - 12^2 - 12^2}{(-2)\times12\times12}$ so $\theta = 77°22'$

b) Area $= \frac{1}{2}bc\sin A = \frac{1}{2}\times12^2\times\sin\theta = 70.3 \text{ cm}^2$

c) Minor segment $= \pi r^2\times\frac{\theta}{360} - 70.3 = \frac{\pi\times12^2}{360}\times\theta - 70.3 = 26.92 \text{ cm}^2$

d) Major segment $= \pi r^2 -$ minor segment

$= \pi\times12^2 - 26.92 = 425.5 \text{ cm}^2$