SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Example 13

Complete the square to solve:

(a) $x^2+4x-5=0$ **(b)** $x^2-5x+6=0$ **(c)** $x^2=8x$

Solution

(a) $x^2+4x-5=0$

$x^2+4x=5$ Add 5 to both sides

$x^2+4x+4=5+4$ Add 4 to both sides to complete the square

$(x+2)^2=9$ Factorise

$x+2=\pm 3$ Take the square roots of each side

$x+2=3$ or $x+2=-3$

$x=1$ or $x=-5$

(b) $x^2-5x+6=0$

$x^2-5x=-6$ Subtract 6 from both sides

$x^2-5x+\frac{25}{4}=-6+\frac{25}{4}$ Add $\frac{25}{4}$ to both sides to complete the square

$\left(x-\frac{5}{2}\right)^2=\frac{1}{4}$ Factorise

$x-\frac{5}{2}=\pm\frac{1}{2}$ Take the square roots of each side

$x-\frac{5}{2}=\frac{1}{2}$ or $x-\frac{5}{2}=-\frac{1}{2}$

$x=3$ or $x=2$

OR, instead of taking the square roots of each side, rewrite the expression as:

$\left(x-\frac{5}{2}\right)^2-\frac{1}{4}=0$

Then, factorise using the difference of two squares:

$\left(x-\frac{5}{2}-\frac{1}{2}\right)\left(x-\frac{5}{2}+\frac{1}{2}\right)=0$

$(x-3)(x-2)=0$

$x=3$ or $x=2$

(c) $x^2=8x$

$x^2-8x=0$ Subtract $8x$ from both sides

$x^2-8x+16=16$ Add 16 to both sides to complete the square

$(x-4)^2=16$ Factorise

$x-4=\pm 4$ Take the square roots of each side

$x-4=4$ or $x-4=-4$

$x=8$ or $x=0$