REGIONS AND INEQUALITIES

A straight line represents a function (unless it is a vertical line). If the equation of a straight line is changed to an inequality, then the function becomes a relation. It is no longer a straight line, but instead it can be represented graphically by a region in the number plane.

To graph a region on the number plane:

- Graph the equation of the region's boundary.
- Select a point not on the boundary.
- Substitute the coordinates of this point into the equation of the boundary.
- If the point makes the inequality true, then all points on that side of the inequality will also make it true. Shade that side of the boundary to indicate the region. (If the point does not make the inequality true, then the points on the other side of the inequality must, so shade that side instead.)

If the inequality includes '… or equal to', then the boundary is part of the region. If the inequality does not include '… or equal to', then the boundary is not part of the region, so it should be dashed to show this.

Example 6

Graph the region in the number plane represented by each inequality.

(a) $x+y\geq 1$ **(b)** $x+y<1$ **(c)** $x+y\leq 1$ **(d)** $x+y>1$

Solution

In each case, first draw the line $x+y=1$. In parts **(b)** and **(d)**, dash the line to show that the boundary is not part of the region. Substitute the non-boundary point $(1, 1)$ into each inequality to find the region side.

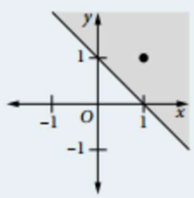
(a) $x+y\geq 1$

Solid line

$2>1$

$\text{LHS}=1+1=2>1$

Result true:
shade above the line



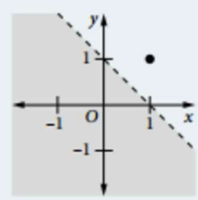
(b) $x+y<1$

Dashed line

$2>1$

$\text{LHS}=1+1=2>1$

Result not true:
shade below the line



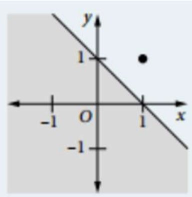
(c) $x+y\leq 1$

Solid line

$2>1$

$\text{LHS}=1+1=2>1$

Result not true:
shade below the line



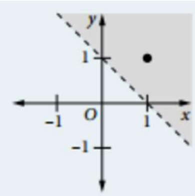
(d) $x+y>1$

Dashed line

$2>1$

$\text{LHS}=1+1=2>1$

Result true:
shade above the line



The regions in **(a)** and **(b)** together make the whole number plane, as do the regions in **(c)** and **(d)** together.

The boundary line divides the number plane into three sets of points: the points on the line, the points above the line and the points below the line. (If the inequality includes '… or equal to', then the boundary is part of the region; if the inequality does not include '… or equal to', then the boundary is not part of the region, so it is dashed.)

Thus for the example above, the region in part **(a)** could be described as 'the set of points on or above the line with equation $x+y=1$'. The region in part **(b)** could be described as 'the set of points below the line with equation $x+y=1$'.

Example 7

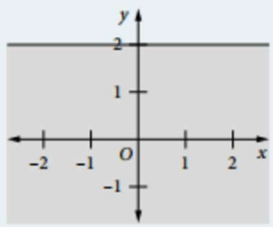
Graph the region in the number plane represented by each inequality.

(a) $y \le 2$ **(b)** $-1 < y \le 2$ **(c)** $x > 1$ **(d)** $x > 1$ or $x \le -1$

Solution

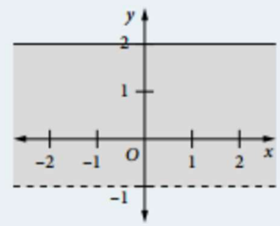
These inequalities have only one variable, so their boundaries are either horizontal lines (as in **(a)** and **(b)**) or vertical lines (as in **(c)** and **(d)**).

(a) $y \le 2$



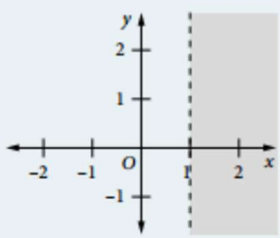
Region is below the line

(b) $-1 < y \le 2$



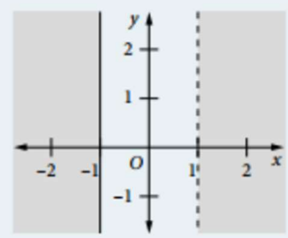
Region is between the lines

(c) $x > 1$



Region is to the right of the line

(d) $x > 1$ or $x \le -1$



Region is outside the lines

Example 8

(a) Sketch the region defined by the intersection $y \geq x^2 - 1$ and $y \leq 3x + 3$.

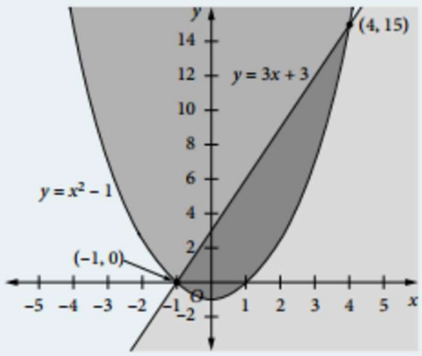
(b) Hence write the solution to $x^2 - 3x - 4 \leq 0$.

(c) Solve $x^2 - 3x - 4 \leq 0$ algebraically to check your solution to **(b)**.

(d) What would be different in this process if you were solving $x^2 - 3x - 4 < 0$?

Solution

(a)



(b) $x^2 - 3x - 4 \leq 0$

Rewrite in terms of given equations: $x^2 - 1 \leq 3x + 3$

Find the points of intersection of the graphs by using simultaneous equations, setting each rule equal to the other: $(-1, 0)$ and $(4, 15)$

Roots of $x^2 - 1 = 3x + 3$ are: $x = -1, 4$

Required region is between the curves: Solution is $-1 \leq x \leq 4$

(c) $x^2 - 3x - 4 \leq 0$

Factorise: $(x + 1)(x - 4) \leq 0$

The roots of $x^2 - 3x - 4 = 0$ are: $x = -1, 4$

Pick a value of x between -1 and 4, e.g. $x = 0$ and substitute into the quadratic expression.

To test $x^2 - 3x - 4 \leq 0$ use $x = 0$.

$x = 0$: $0^2 - 3(0) - 4 \leq 0$

$-4 \leq 0$

Since this value makes the inequality true, it must lie in the region defined where $-1 \leq x \leq 4$.

(d) Graphically, the boundary would not be included so the parabola and the lines would be dashed. The solution would not include equality, it would be $-1 < x < 4$.