THE NORMAL DISTRIBUTION

1 The time taken for all competitors to finish the 50 m freestyle at the school swimming carnival, X seconds, was found to follow a normal distribution where $X \sim N(45, 9)$. Find the following values.

(a) The time range in which you would expect to find the middle 95% of results.
(b) The percentage of students you would expect to take more than 48 seconds.
(c) The percentage of students you would expect to take less than 36 seconds.

a) 95% of observations would be in the range $\mu \pm 2\sigma$

Here $\mu = 45$ and $\sigma = \sqrt{9} = 3$

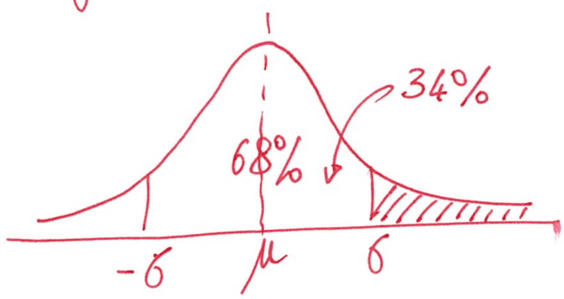
So 95% of observations would be within the range $45 \pm 3 \times 2$

so 45 ± 6, so from 39 to 51

b) $48 = 45 + 3 = \mu + \sigma$

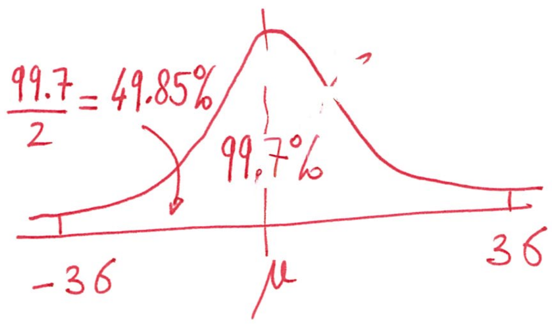
So we look for the proportion of students whose time is more than $(\mu + \sigma)$

So it's 16%



c) $36 = 45 - 9 = \mu - 3\sigma$

So about 0.15% of students would take less than 36 s.



2 $X \sim N(10, 4)$.

(a) What is the range of x values in which you would expect to find the middle 68%?

(b) What is the range of x values in which you would expect to find the middle 95%?

(c) What is the range of x values in which you would expect to find the middle 99.7%?

a) within $10 \pm \sigma$

Here $\sigma^2 = 4$ so $\sigma = 2$

So between 8 and 12

b) within $10 \pm 2\sigma$ so between 6 and 14

c) — $10 \pm 3\sigma$ so between 4 and 16

THE NORMAL DISTRIBUTION

3 The height X cm of a population is known to be distributed as $X \sim N(170, 81)$.

(a) What is the percentage of the population expected to be found in the range 152–188 cm?
(b) What is the percentage of the population expected to be taller than 197 cm?
(c) What is the percentage of the population expected to be shorter that 170 cm?
(d) What is the percentage of the population expected to be found in the range 161–197 cm?

$\sigma^2 = 81$ so $\sigma = 9$

a) $152 = 170 - 18 = 170 - 2\sigma$
$188 = 170 + 18 = 170 + 2\sigma$
So 95% of the population would be between 152 and 188.

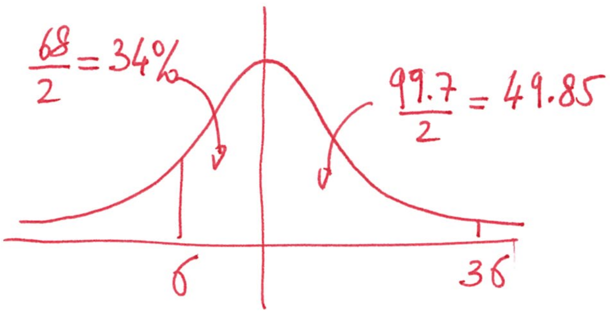
b) $197 = 170 + 27 = 170 + 3\sigma$

$\frac{99.7}{2} = 49.85$ so only 0.15% of the population is taller than 197 cm

c) $170 = \mu$. so 50%

d) $161 = 170 - 9 = 170 - \sigma$

So $34 + 49.85 = 83.85\%$



4 The graph represents a continuous random variable which has a normal distribution.

The distribution is best represented by:

A $N(14, 26)$ B $N(26, 12)$ $\boxed{C}$ $N(20, 9)$ D $N(20, 3)$

$26 = \mu + 2\sigma$
$14 = \mu - 2\sigma$

So $\mu = \frac{14+26}{2} = 20$ and $2\sigma = 6$ so $\sigma = 3$

Reponse $\boxed{C}$ $\sigma^2 = 9$

THE NORMAL DISTRIBUTION

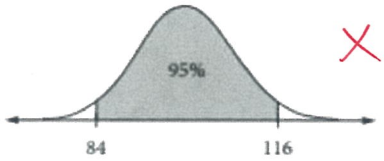
so $\sigma^2 = 16$ then $\sigma = 4$

5 Given X is normally distributed with a mean of 100 and a variance of 16, which of the following graphs correctly represents the distribution?

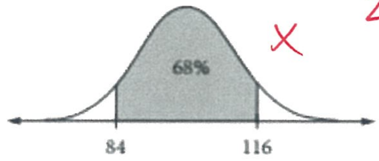
$2\sigma = 8$

~~so far~~

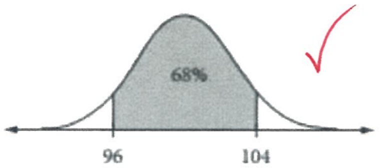
A



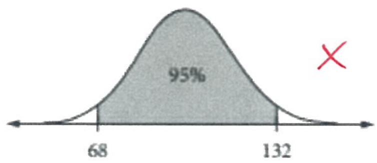
C



B



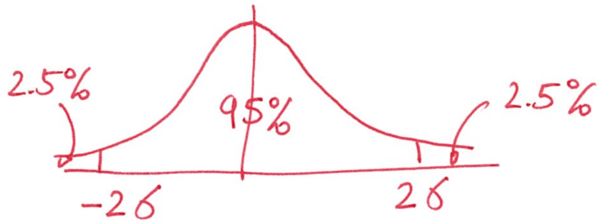
D



6 The marks X obtained by students in an examination were normally distributed with a mean of 85 and a standard deviation of 4. If the top 2.5% of students received a prize, find the minimum whole number score possible to receive a prize.

$\mu = 85 \quad \sigma = 4$

$2\sigma = 8$



So the minimum whole number score possible to receive a prize is

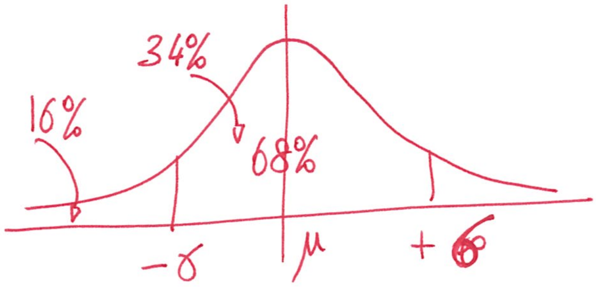
$\mu + 2\sigma = 85 + 8 = 93$

THE NORMAL DISTRIBUTION

7 The mass M grams of a batch of commemorative coins is such that $M \sim N(50, 9)$. Each coin is weighed before packaging and will be rejected if its mass is less than 47 g. What is the percentage of coins expected to be rejected?

$\mu = 50 \quad \sigma^2 = 9$ so $\sigma = 3$

$47 = 50 - 3 = \mu - \sigma$

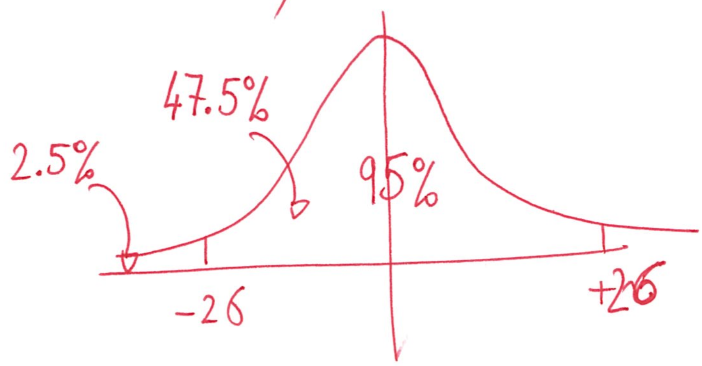


So 16% of coins are expected to be rejected.

8 Packets of 'Greatstart' breakfast cereal are labelled as having a mass of 500 g. However, the machine that fills the packets actually follows a normal distribution with a mean of 510 g and a standard deviation of 5 g. What percentage of packets, correct to two decimal places, will have a mass less than 500 g?

$\mu = 510g \quad \sigma = 5g.$

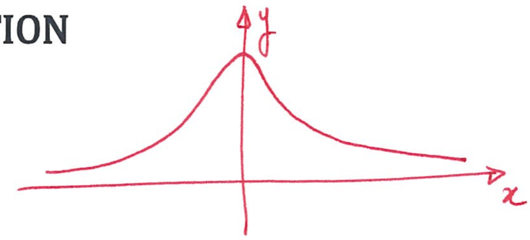
$500 = 510 - 10 = 510 - 2 \times 5 = \mu - 2\sigma$



So 2.5% of packets are expected to have a mass less than 500g.

THE NORMAL DISTRIBUTION

12 (a) Using graphing software draw the graph of $f(x) = e^{-x^2}$.



(b) Use Desmos to estimate $\int_{-4}^{4} e^{-x^2}\,dx$

$$\int_{-4}^{4} e^{-x^2}\,dx = 1.77245\ldots$$

13 (a) Using graphing software, draw the graph of $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, for:

(i) $\mu = 10, \sigma = 3$

(ii) $\mu = 0, \sigma = 1$

(b) Use the integration tool in the software to evaluate $\int_{\mu-3\sigma}^{\mu+3\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$ in each case in part **(a)**.

a) $\mu = 10$, $\sigma = 3$ gives $f(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-10)^2}{18}}$

$\mu = 0$, $\sigma = 1$ gives $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

b) $$\int_{1}^{19} \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-10)^2}{18}}\,dx \simeq 0.997294$$

$$\int_{-3}^{3} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx \simeq 0.997300$$