APPLICATIONS OF CALCULUS - CHAPTER REVIEW

1 By integration, find the volume of the solid of revolution formed from the region bounded by:

(a) the circle $x^2 + y^2 = 1$, rotated about the x-axis

(b) the line $y = x + 3$ between $x = 0$ and $x = 2$, rotated about the x-axis

(c) the parabola $y = x^2 + 3$ between $y = 4$, $y = 12$ and the y-axis, rotated about the y-axis.

3 Using the substitution $u=\sqrt{x}$, find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$.

4 **(a)** Use the substitution $u=1-x$ to evaluate $\int_0^1 2x\sqrt{1-x}\,dx$.

5 Using the substitution $u = x^3 + 1$, or otherwise, evaluate $\int_0^1 x^2 e^{x^3+1}\,dx$.

6 Evaluate $\int_0^{\frac{\pi}{4}} \sin\theta\cos^2\theta\,d\theta$.

7 Use the substitution $u = \log_e x$ to evaluate $\int_e^{e^3} \frac{1}{x(\log_e x)^2} dx$.

11 **(a)** Differentiate $e^{2x}(2\sin x - \cos x)$. **(b)** Hence, or otherwise, find $\int e^{2x} \sin x \, dx$.

14 **(a)** By expanding the left-hand side, show that $\sin(6x + 3x) + \sin(6x - 3x) = 2\sin 6x \cos 3x$.

(b) Hence find $\int \sin 6x \cos 3x \, dx$.

15 Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = \sin 2x$, the x-axis and the line $x = \frac{\pi}{6}$ is rotated about the x-axis.