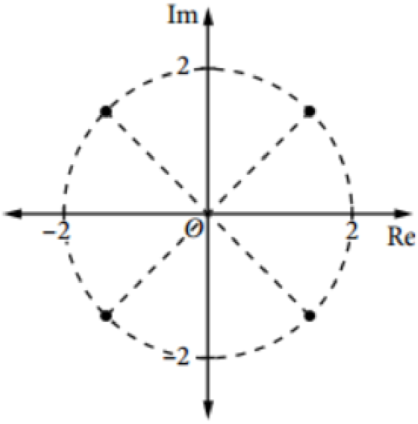
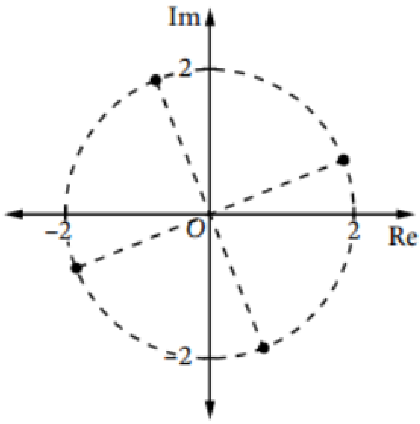
ROOTS OF COMPLEX NUMBERS

1 Which Argand diagram best shows the fourth roots of $16i$?

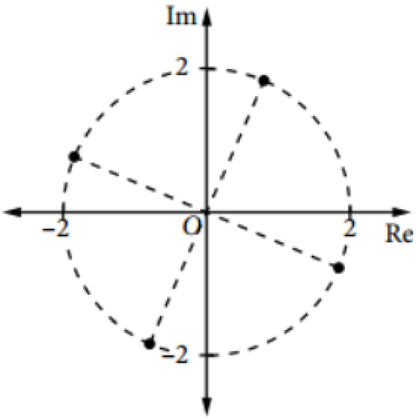
A



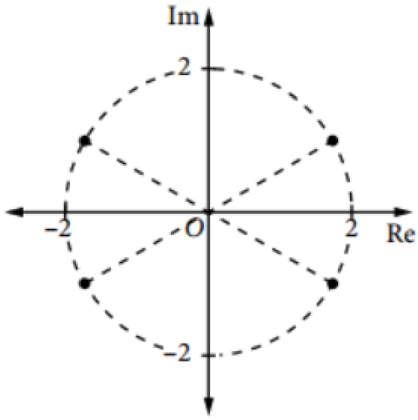
B



C



D



2 For each of the following, find the values of z (in mod–arg form) and plot them on the complex plane.

(a) $z^5 = 1$ **(b)** $z^4 + 1 = 0$ **(c)** $z^2 = i$ **(d)** $z^3 + 8i = 0$ **(e)** $z^4 = 8(\sqrt{3} + i)$ **(f)** $z^6 = i$

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4 The point $1 + \sqrt{3}i$ and two other points are on the circumference of a circle with centre O and radius 2. The three points are the vertices of an equilateral triangle.

(a) Find the complex numbers represented by the two other points.

(b) Find the cubic equation that has these three complex numbers as its roots.

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5 If 1, w_1 and w_2 are the cube roots of unity, prove the following:

(a) $w_1 = \overline{w_2} = w_2^2$ **(b)** $w_1 + w_2 = -1$ **(c)** $w_1 w_2 = 1$

6 If w is a non-real cube root of unity (i.e. w is a non-real root of $z^3 = 1$), show the following:

(a) $1 + w + w^2 = 0$ **(b)** $(1 - w)(1 - w^2) = 3$

Now evaluate the following:

(c) $(1 + w)^3$ **(d)** $(1 + 2w + 3w^2)(1 + 2w^2 + 3w)$ **(e)** $(w^2 + 2w + w^3)(2w^2 + w + w^3)$

(f) $(1 - w)(1 - w^2)(1 - w^4)(1 - w^5)(1 - w^7)(1 - w^8)$

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8 **(a)** Find the roots of $z^7 = 1$ in mod–arg form and show them on an Argand diagram.

(b) If w is a non-real root, show that $w + w^2 + w^3 + w^4 + w^5 + w^6 = -1$.

(c) Show that the quadratic equation $z^2 + z + 2 = 0$ has roots $w + w^2 + w^4$ and $w^3 + w^5 + w^6$.

(d) Show that $\cos\frac{\pi}{7} = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \frac{1}{2}$.

11 **(a)** Show that $z_1 = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ is a root of $z^4 + z^3 + z^2 + z + 1 = 0$.

(b) Find all four roots of $z^4 + z^3 + z^2 + z + 1 = 0$.

(c) Show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$.

(d) Deduce that $\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}$.