SOLVING SIMULTANEOUS EQUATIONS - LINEAR AND SECOND DEGREE

Solve the following simultaneous equations. (Try to solve some using graphing software.)

1 $y = 5x + 6$, $y = x^2$

2 $y = 3x - 2$, $y = x^2$

3 $y = x + 5$, $y = x^2 - 3x$

4 $x + y = 15$, $y = x^2 - 6x + 1$

1) $\Leftrightarrow x^2 = 5x + 6 \Leftrightarrow x^2 - 5x - 6 = 0$ $\Delta = 25 + 4 \times 6 = 49 = 7^2$

$\begin{cases} x_1 = \frac{5+7}{2} = 6 \\ y_1 = 36 \end{cases}$ or $\begin{cases} x_2 = \frac{5-7}{2} = -1 \\ y_2 = 1 \end{cases}$

2) $x^2 - 3x + 2 = 0$ $\Delta = 9 - 4 \times 2 = 1$

$\begin{cases} x_1 = \frac{3+1}{2} = 2 \\ y_1 = 4 \end{cases}$ or $\begin{cases} x_2 = \frac{3-1}{2} = 1 \\ y_2 = 1 \end{cases}$

3) $x^2 - 3x = x + 5 \Leftrightarrow x^2 - 4x - 5 = 0$ $\Delta = 16 + 20 = 36$

$\begin{cases} x_1 = \frac{4+6}{2} = 5 \\ y_1 = 10 \end{cases}$ or $\begin{cases} x_2 = \frac{4-6}{2} = -1 \\ y_2 = 4 \end{cases}$

4) $x + (x^2 - 6x + 1) = 15 \Leftrightarrow x^2 - 5x - 14 = 0$

$\Delta = 25 + 4 \times 14 = 81 = 9^2$

$\begin{cases} x_1 = \frac{5-9}{2} = -2 \\ y_1 = 15 - (-2) = 17 \end{cases}$ OR $\begin{cases} x_2 = \frac{5+9}{2} = 7 \\ y_2 = 15 - 7 = 8 \end{cases}$

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9 $x-y=1$, $xy=2$ $\quad y=x-1$

10 $y=2x-5$, $y=x^2-4x+4$

11 $y=2x-6$, $x^2-xy+2y^2=16$

12 $x+y=5$, $3x^2+xy-y^2=29$

9) $x(x-1)=2 \iff x^2-x-2=0 \qquad \Delta = 1+4\times 2 = 9 = 3^2$

$$\begin{cases} x_1 = \frac{1-3}{2} = -1 \\ y_1 = -1-1 = -2 \end{cases} \quad \text{OR} \quad \begin{cases} x_2 = \frac{1+3}{2} = 2 \\ y_2 = 2-1 = 1 \end{cases}$$

10) $x^2-4x+4 = 2x-5 \iff x^2-6x+9=0$

$\Delta = 36-4\times 9 = 0 \qquad \begin{cases} x = \frac{4}{2} = 2 \\ y = 2\times 2 - 5 = -1 \end{cases}$

11) $x^2 - x(2x-6) + 2(2x-6)^2 = 16$

$\iff x^2 - 2x^2 + 6x + 2(4x^2-24x+36) = 16$

$\iff -x^2 + 6x + 8x^2 - 48x + 72 = 16$

$\iff 7x^2 - 42x + 56 = 0 \iff x^2 - 6x + 8 = 0$

$\Delta = 36 - 4\times 8 = 4 = 2^2 \qquad \begin{cases} x_1 = \frac{6-2}{2} = 2 \\ y_1 = 2\times 2 - 6 = -2 \end{cases} \quad \text{OR} \quad \begin{cases} x_2 = \frac{6+2}{2} = 4 \\ y_2 = 2\times 4 - 6 = 2 \end{cases}$

12) $3x^2 + x(5-x) - (5-x)^2 = 29$

$\iff 3x^2 + 5x - x^2 - (25 - 10x + x^2) = 29$

$\iff 2x^2 + 5x - 25 + 10x - x^2 = 29$

$\iff x^2 + 15x - 54 = 0 \qquad \Delta = 15^2 + 4\times 54 = 441 = 21^2$

$$\begin{cases} x_1 = \frac{-15-21}{2} = -18 \\ y_1 = 5-(-18) = 23 \end{cases} \quad \text{OR} \quad \begin{cases} x_2 = \frac{-15+21}{2} = 3 \\ y_2 = 5-3 = 2 \end{cases}$$

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17 $x+y=2$, $x^2+y^2=2$ 18 $x=2y-1$, $3x^2=x+2y^2$ 19 $x+2y=-8$, $xy=8$ 20 $y=x+9$, $y=x^2-x-6$

17) $x^2+(2-x)^2=2 \iff 2x^2-4x+4=2 \iff 2x^2-4x+2=0$

$x^2-2x+1=0 \iff (x-1)^2=0$ $\boxed{x=1}$ $y=2-1=1$

18) $3(2y-1)^2=2y-1+2y^2 \iff 3(4y^2-4y+1)-2y+1-2y^2=0$

$\iff 12y^2-12y+3-2y+1-2y^2=0$

$\iff 10y^2-14y+4=0 \iff 5y^2-7y+2=0$

$\Delta=49-4\times5\times2=9=3^2$

$\begin{cases} y_1=\frac{7-3}{10}=+\frac{4}{10}=\frac{2}{5} \\ x_1=2\times\frac{2}{5}-1=-\frac{1}{5} \end{cases}$ OR $\begin{cases} y_2=\frac{7+3}{10}=1 \\ x_2=2\times1-1=1 \end{cases}$

19) $x=-8-2y$ so $(-8-2y)y=8 \iff -2y^2-8y-8=0$

$\iff y^2+4y+4=0$

$\Delta=16-4\times4=0$

$y=\frac{-4}{2}=-2$ and $x=-8-2\times(-2)=-4$

20) $x+9=x^2-x-6 \implies x^2-2x-15=0$

$\Delta=4-4\times(-15)=64=8^2$

$\begin{cases} x_1=\frac{2-8}{2}=-3 \\ y_1=-3+9=6 \end{cases}$ OR $\begin{cases} x_2=\frac{2+8}{2}=5 \\ y_2=5+9=14 \end{cases}$